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Optimizing the Fraction of Expensive Direct Measurements in an Exposure Assessment Study

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Abstract: When designing studies to assess occupational exposures, one persistent decision problem is the selection between two technical methods, where one is expensive and statistically efficient and the other is cheap and statistically inefficient. While a few studies have attempted to determine the relatively more cost-efficient design between two technical methods, no successful study has optimized the fraction of the expensive efficient method in a combined technique intended for long-run exposure assessment studies. The purpose of this study was therefore to optimize the fraction of the expensive efficient measurements by resolving a precision-requiring cost minimization problem. For an indefinite total number of measurements, the total cost of a working posture assessment study was minimized by performing only expensive direct technical measurements. However, for a definite total number of measurements, the use of combined techniques in assessing the posture could be optimal, depending on the constraints placed on the precision and on the research budget.

Keyword: Statistical efficiency, combined measurement technique, productive efficiency, cost savings, marginal cost-benefit ratio, cost function.

1. INTRODUCTION

An important decision to make when planning an assessment study of occupational exposures is choosing between a ‘cheap’ (resource-saving) subjective method and an ‘expensive’ (resource-using) direct technical method among the many techniques that have been developed [1-7]. Subjective methods may be selected due to constrained research budgets, while the direct technical methods are usually chosen to satisfy a requirement of *statistical efficiency* (i.e. the ability to produce minimum variance unbiased estimates). In general, indirect subjective measurement methods are known to be less costly and more feasible [8, 9], but produce measurement error [10, 11]. Conversely, direct technical measurement methods are known to produce large volumes of good-quality (low-error) quantitative data on exposure variables, but at a higher cost [3, 12]. A similar decision problem may appear in choosing between a cheap simple technique and an expensive advanced technique when assessing the exposure data recorded by a video-based observation method [13]. Another economic decision problem when designing an exposure assessment study could be the choice between skilled or unskilled investigators to record/analyse exposure data; differing skill levels could produce differences in cost and even statistical efficiency [14-17].

In all the examples above, decision problems only appear when the advanced measurement technique is both economically expensive and statistically efficient compared to the simple assessment technique. There is thus no problem when making a decision about an alternative technical method that is expensive but produces the same or a lower quality of estimates on an exposure, provided that both techniques are feasible and appropriate in assessing the exposure. Analytical tools are available for comparing alternative exposure measurement designs according to their ‘economic cost’ and ‘statistical efficiency’ when the objectives are: 1) to determine the highest point at which more investment in increased precision is worthwhile [18, 19]; 2) to identify the measurement design(s) providing higher precision for a particular number of measurements, or the design(s) requiring fewer exposure measurements for an acceptable precision [20, 21]; and 3) to measure relative cost efficiencies of alternative assessment techniques in terms of their ability to produce information at low cost [13]. To summarize, the relevant studies attempt to compare *cost-efficiency* in the ‘alternative’ measurement designs in order to select the *right* design for exposure measurement [22]. However, there is no widely accepted and formal definition of cost-efficiency in the economic evaluation of ‘non-optimal alternative measurement designs’ (i.e. different designs that can be implemented in an exposure assessment study; none of them is optimized in its economic cost and/or statistical efficiency) One could, cautiously, say that the measurement design that produces less error on exposure relative to the cost of achieving it is the

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most satisfactory measurement design to employ [13]. There are additional limitations in this approach to cost-efficiency analysis. Analytical tools intended for use in the comparison approach do not provide evidence on technical efficiency (e.g. ability to maximize precision for the given economic resources) and productive efficiency (e.g. ability to achieve a predetermined precision at minimum cost) of the kind that an optimization approach would provide. Methodologies for comparing the cost-efficiency of alternative non-optimal measurement designs are only analytical tools to help decision makers allocate the available resources in a way that reduces statistical error or economic cost. Only under the assumption of the same returns *to* scale (or economies of scale) for alternative designs will the analytical tools be able to identify the relatively cost-efficient technique. Hence, 'returns to scale' and 'economies of scale' are related economic terms that describe, for instance, what happens as the scale of information output increases in the long run, i.e. when all measurement inputs are variable.

The currently used measurement methods (i.e. self-reports, observation, and direct technical methods) are not only different in economic cost and statistical efficiency, but also in feasibility and appropriateness. As each of the methods is capable of measuring one or more aspects of exposures, while has drawbacks as well [1], a method can be combined with others in an exposure assessment study [3]. Considering this, the methods are combined to remedy the disadvantages associated with each [23]. The introduced measurement methods can thus be complementary to each other in attempting to produce information on work-related exposures [24, 25]. The combined approach may also be appropriate when there are technological constraints in, for example, competences, technical support, and working conditions, particularly around direct technical measurements of work-related exposures. Among all measurement methods, direct technical and observation methods have been simultaneously used in assessment of work postures [13, 24]. The information produced by them can thus be combined in exposure assessments for statistical, technological and practical reasons. Direct measurements may be added in an attempt to increase accuracy, while observational assessments may be retained for several reasons. Firstly, the information provided by observation may complement the directly measured data in a way that helps the researcher gain a more complete and explicative picture of the exposure. Secondly, observational assessments may be more suitable given the technological and practical constraints in competences and time available for analysis of the data produced by the direct technical method. Finally, the availability of technical support and the preference of participants for observational assessments may also affect the decision to include indirect assessments. The combined approach have also been used in "powerful" exposure-response studies, where both "perfect" and "imperfect" measurement methods are used to assess a continuous exposure variable [26, 27].

The interesting research question is whether the economic and statistical performance of a combined approach can be optimized. According to optimization approach, a combination of expensive direct and cheap subjective methods in larger exposure studies [23] can be advantageous compared to the choice of a relatively cost-efficient measurement method [18, 19]. The combined measurement technique to estimate/predict group mean exposures may be preferred because of its potential capacity to save total cost and further to reduce the measurement bias associated with indirect estimates or the selection bias associated with direct technical measurements [23]. When the average costs and statistical efficiencies associated with two alternative measurement methods are known, a determined budget can be used to identify the method that gives better statistical efficiency at the same cost. If, however, a combined technique is considered due to the abovementioned requirements and constraints, its ability to produce information at low cost should be optimized. While the optimal technique for producing exposure data may not be identified by applying analytical tools in the comparison approach, it may be identified by combining the two alternative technical methods. Hence, one interesting research challenge in relation to exposure assessment studies is to find a potential combined measurement technique that increases the cost-efficiency of a single measurement technique. Theoretically, optimizing resource allocation between two alternative measurement techniques (ibid.) would increase the cost-efficiency provided by the relatively cost-efficient assessment technique [18, 19]. However, only one study, that by Duan and Mage (1997), has attempted to optimize the fraction of expensive direct technical measurements, and it was not entirely successful. The assumption of a correlation between direct and indirect measurements, and the attempt to use the estimated correlation to predict direct measurements that were not in fact performed, resulted in optimizing a fraction of 'dual sample', in which both methods are used. The dual sample approach exhibited an additional strong assumption that a combined measurement technique was always the optimal choice. The total number of measurements, another important variable in using any combined technique, was not determined in the model. Thus, the influence of the budget constraint defined in the optimization problem on a possible combined technique was not known. Finally, the optimized fraction of dual sample was unfortunately allowed to exceed unity [22].

The constrained optimization problems in any exposure assessment study are either maximizing the precision of the assessments at a given total cost or minimizing the total cost of the assessments for a desired level of precision. An optimal fraction of expensive direct measurements may be derived by resolving each of the optimization problems, because of their duality property. However, when attempting to optimally allocate resources for exposure assessment studies, the objective of minimizing costs involves weaker assumptions than the objective of maximizing precision. Thus, like in any production, it seems more rational to choose a design that saves the cost of producing a certain level of output (in this case, the precision of a combined mean exposure estimate).

The purpose of this study was to economically evaluate different fractions of the expensive direct technical measurements based on the precision-requiring cost minimization approach. Providing an optimal combined technique can be an alternative way to select a relatively cost-efficient technique in exposure assessment studies. When the economic and technological resources for direct technical measurements of an occupational exposure are limited, and the statistical efficiency of exposure assessments yielded by subjective methods is not satisfactory, the ‘optimal’ measurement technique may be one that combines the two ‘non-optimal’ methods.

2. METHODS

To minimize the cost of achieving a required precision by using a combined assessment technique, two functions are required: 1) a *constraint* function to assess the precision of the combined technique, and 2) an *objective* function to calculate the total cost of using the technique. To derive the constraint function, the combined mean and the variance of the combined mean should be formulated. Thus, we began our analysis by assuming that direct technical measurements and indirect subjective estimates are *composite inputs* to an exposure assessment study, with the overall exposure mean using the assumed combined measurement technology estimated as follows:

$$\hat{\mu}_c = f_1\hat{\mu}_1 + f_2\hat{\mu}_2, \tag{1}$$

where $\hat{\mu}_c$ is the combined exposure mean and $\hat{\mu}_1$ and $\hat{\mu}_2$ stand for exposure mean estimates of direct technical measurements and indirect subjective estimates, respectively; and f_1 and f_2 denote the fractions of the direct and subjective measurements, respectively. If n_1, n_2, and N are the numbers of direct, indirect, and total measurements, respectively, the fractions of direct technical measurements and subjective estimates can be calculated as follows:

$$f_1 = \frac{n_1}{N};\ f_2 = \frac{n_2}{N}, \tag{2}$$

Where $N = n_1 + n_2$ and $f_1 + f_2 = 1$.

Assuming that the two techniques independently measure/estimate exposure, and no type of bias is known, the variance of the combined mean exposure, $Var(\hat{\mu}_c)$, can then be calculated as follows:

$$Var(\hat{\mu}_c) = f_1^2\frac{\hat{\sigma}_1^2}{n_1} + f_2^2\frac{\hat{\sigma}_2^2}{n_2}, \tag{3}$$

Where $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$ are the mean variances estimated by the direct measurement technique and the indirect subjective method, respectively.

We substitute (2) into (3) and rearrange terms in order to estimate the variance by using N instead of n_1 and n_2:

$$Var(\hat{\mu}_c) = \frac{f_1\hat{\sigma}_1^2 + f_2\hat{\sigma}_2^2}{N} \tag{4}$$

The precision of the combined measurement technique (P), which is the constraint of the optimization problem, can be estimated as the inverse of the standard error of the combined mean:

$$P = \frac{1}{\sqrt{Var(\hat{\mu}_c)}} = \left(\frac{f_1\hat{\sigma}_1^2 + f_2\hat{\sigma}_2^2}{N}\right)^{-\frac{1}{2}} \tag{5}$$

The total cost (TC) of the combined technique, which is the objective function of the optimization problem, can be expressed as follows:

$$TC = n_1\bar{c}_1 + n_2\bar{c}_2 \,, \tag{6}$$

where $\bar{c}_1$ and $\bar{c}_2$ are the average unit prices of a direct measurement and an indirect subjective estimate, respectively.

We substitute (2) into (6) and rearrange terms in order to correlate the total cost to *N* and the fractions that are shared by the precision equation (5):

$$TC = N \cdot \left(f_1\bar{c}_1 + f_2\bar{c}_2 \right) \tag{7}$$

Resolving the precision equation (5) and the cost equation (7) for *N*, we can estimate *N* as a function of precision and total cost, respectively:

$$N = P^2 \cdot \left(f_1\hat{\sigma}_1^2 + f_2\hat{\sigma}_2^2 \right) \tag{8}$$

$$N = \frac{TC}{f_1\bar{c}_1 + f_2\bar{c}_2} \tag{9}$$

By (8) and (9), the total cost of the combined technique can be calculated in relation to the precision, the variances estimated by the techniques, and the average unit prices for different fractions of direct measurements:

$$TC = P^2 \cdot \left[f_1\bar{c}_1 + \left(1 - f_1\right)\bar{c}_2 \right] \cdot \left[f_1\hat{\sigma}_1^2 + \left(1 - f_1\right)\hat{\sigma}_2^2 \right] \tag{10}$$

Two cost curves are used to evaluate the combined technique economically: *average cost* (AC) and *marginal cost* (MC). AC is the cost per unit output, and is obtained here by dividing the total cost function by the precision; MC is the cost associated with producing one additional unit of the output, and is obtained by differentiating the total cost function with respect to precision as follows:

$$AC = \frac{TC}{P} = P \cdot \left[f_1\bar{c}_1 + \left(1 - f_1\right)\bar{c}_2 \right] \cdot \left[f_1\hat{\sigma}_1^2 + \left(1 - f_1\right)\hat{\sigma}_2^2 \right] \tag{11}$$

$$MC = \frac{\partial TC}{\partial P} = 2P \cdot \left[f_1\bar{c}_1 + \left(1 - f_1\right)\bar{c}_2 \right] \cdot \left[f_1\hat{\sigma}_1^2 + \left(1 - f_1\right)\hat{\sigma}_2^2 \right] \tag{12}$$

The cost function (10) shows that the combined measurement technique can be characterized by *diseconomies of scale* (cost disadvantage to improve the precision), since the *cost elasticity of precision* (E_P^C), which shows the percentage change in cost as a result of a one percent change in the precision, exceeds unity:

$$E_P^C = \frac{\partial \ln TC}{\partial \ln P} = \frac{\partial TC}{\partial P} \cdot \frac{P}{TC} = \frac{\partial TC}{\partial P} \Big/ \frac{TC}{P} = \frac{MC}{AC} = 2 \tag{13}$$

Thus, an improvement in precision by one percent requires a two percent increase in the cost. The statistical production technology exhibits *decreasing returns to scale* (i.e. the amount of improvement in precision is less than the amount of proportional increase in n_1 and n_2), since

$$E_C^P = 1 \Big/ \frac{\partial \ln C}{\partial \ln P} = 1/2 = 0.5 < 1 \,.$$

Regression technique and equation (10) are used to estimate/predict two important costs associated with the combined technique: 1) the cost of achieving a required precision (TC_{P_0}) at each fraction of the direct measurements by using the following regression equation:

$$TC_{P_0} = \alpha + \beta_1 \cdot f_1 + \beta_2 \cdot f_1^2 \,, \tag{14}$$

and 2) the cost of each level of precision while the fractions are constants (TC_{f_0}), by using the following regression equation:

$$TC_{f_0} = a + b_1 \cdot P + b_2 \cdot P^2 \tag{15}$$

Hence intercept *α* in (14) (or *a* in (15)) is the autonomous cost, which is not dependent on f_1 (or *P* in (15)); slopes β_1 and β_2 in (14) (or b_1 and b_2 in (15)) show the rates at which the total cost is changed when f_1 (or *P* in (15)) increases by one unit. The use of *quadratic* regression equations is due to non-linearity in *TC* relationships with *P* and $f_1$shown by equation (10).

The productive efficiency (PE) of a non-optimal alternative is obtained by dividing the cost of the optimal choice by the cost of the non-optimal alternative when the precision is the same. The *cost saving* (CS) from the elimination of productive inefficiency associated with each non-optimal design is then estimated as $1 - \mathrm{PE}$.

The cost of improving precision by one unit in using the more precise design *j* is estimated by using the term *marginal cost-benefit ratio* (MCBR) defined as the incremental cost of the design divided by its incremental precision, compared to a cheaper and less precise design *i*:

$$MCBR_{(j)} = \frac{TC_{(j)} - TC_{(i)}}{P_{(j)} - P_{(i)}} \tag{16}$$

Table 1: Calculation of Costs (TC, AC, MC), Total Number of Measurements (N), Numbers of Direct Measurements (n_1) and Indirect Estimates (n_2), Productive Efficiency (PE), and Cost Saving (CS) for Different Values of f_1 and a Predetermined Precision (P). Costs are in SEK, and all Values are Rounded to the Nearest Integer

f_1	$1-f_1$	P	TC	AC	MC	N	n_1	n_2	PE (%)	CS (%)
0.00	1.00	2.75	264967	96352	192704	346	0	346	41	59
0.05	0.95	2.75	267847	97399	194798	333	17	316	41	59
0.10	0.90	2.75	269607	98039	196077	319	32	287	41	59
0.15	0.85	2.75	270245	98271	196542	305	46	259	41	59
0.20	0.80	2.75	269762	98095	196190	291	58	233	41	59
0.25	0.75	2.75	268158	97512	195024	277	69	208	41	59
0.30	0.70	2.75	265433	96521	193042	263	79	184	41	59
0.35	0.65	2.75	261587	95123	190245	249	87	162	42	58
0.40	0.60	2.75	256620	93316	186633	236	94	141	43	57
0.45	0.55	2.75	250532	91103	182205	222	100	122	44	56
0.50	0.50	2.75	243323	88481	176963	208	104	104	45	55
0.55	0.45	2.75	234994	85452	170904	194	107	87	47	53
0.60	0.40	2.75	225543	82016	164031	180	108	72	49	51
0.65	0.35	2.75	214971	78171	156343	166	108	58	51	49
0.70	0.30	2.75	203278	73919	147839	153	107	46	54	46
0.75	0.25	2.75	190464	69260	138520	139	104	35	58	42
0.80	0.20	2.75	176530	64193	128385	125	100	25	62	38
0.85	0.15	2.75	161474	58718	117436	111	94	17	68	32
0.90	0.10	2.75	145297	52835	105671	97	88	10	75	25
0.95	0.05	2.75	127999	46545	93090	83	79	4	86	14
1.00	0.00	2.75	109581	39848	79695	70	70	0	100	0

3. A PRACTICAL EXAMPLE

Data

The data for this example are drawn from a study by Rezagholi *et al*. (2012), who included an investigation of the proportion of time that hairdressers worked with their upper arm above 60° during a two-hour period. The average cost for a two hours' simple observation-based assessment was estimated at $\bar{c}_2 = 765$ SEK[1], with a random error of $\hat{\sigma}_2^2 = 45.8$ including the observer-based variances produced by the technique. Further, the average cost for two hours' direct measurements of the posture by inclinometer was estimated at $\bar{c}_1 = 1575$ SEK, with a random error of $\hat{\sigma}_1^2 = 9.2$. We can conclude that the expensive direct measurements are more cost-efficient compared to the observational assessments, when the cost ratio $\bar{c}_1/\bar{c}_2$ is less than the variance ratio $\hat{\sigma}_2^2/\hat{\sigma}_1^2$.

Constraints

For the first scenario of cost calculation, with a predetermined level of precision and an indefinite number of measurements, the precision is set to 2.75; while for the second scenario, with a predetermined number of measurements ($N = 100$) due to a limited number of workers participating in the exposure assessment study, both precision and cost are calculated for each fraction of direct measurements. The fraction of expensive direct measurements by inclinometer is bounded between zero and unity; that is, $0 \le f_1 \le 1$.

4. EMPIRICAL RESULT

4.1. Indefinite Number of Total Measurements

The total cost of statistical production for achieving a certain level of precision changes as the fraction of

[1]EUR and USD to SEK exchange rate, 3 June 2011, were 9.00 and 6.14, respectively.

direct measurements is increased (cf. Table **1**). The relationship between total cost and f_1 is shown in Figure **1**.

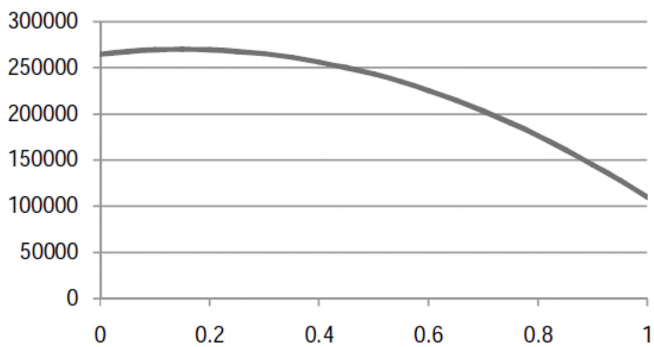

Figure 1: Total cost of achieving the required precision as a function of f_1.

As shown in Table **1** and Figure **1**, the total cost of achieving the required precision initially increases at a decreasing rate as the fraction of direct measurements increases from $0 \to 0.15$. However, from $0.15 \to 1$, the total cost decreases at an increasing rate, which means that the cost of the combined technique can be decreased while retaining precision, by increasing f_1. The cost of the statistical production for achieving the required precision is lowest when $f_1 = 1$; in other words, the empirical data indicate that it is cheaper to produce information by using the direct technical method rather than the observation method. This means that the fraction of observational assessments should be limited to the lowest possible level for any combined assessment technique.

Analysis of the regression model (14) gives the following result:

$$TC_{P=2.75} = 264967 + 68811 \cdot f_1 - 224198 \cdot f_1^2 \quad (17)$$

Hence the autonomous cost of the statistical production in the regression equation (264967 SEK) is the cost of achieving the required precision by using only the observation method (cf. Table **1**). The high value of β_2 shows the significant curvature of the cost function, while its negative sign shows that the cost decreases at an increasing rate as the fraction of direct measurements is increased.

When the fraction of the direct measurements is predetermined due to practical and/or technological constraints, the total cost of statistical production grows at an accelerating rate as the level of precision is increased, because the marginal cost of output (precision) is twice the average total cost (cf. equation (13)). The estimated regression model (15) for $f_1 = 0.5$ is:

$$TC_{f_1=0.5} = 32175 \cdot P^2 \quad (18)$$

The relationship between total cost and precision is shown in Figure **2**.

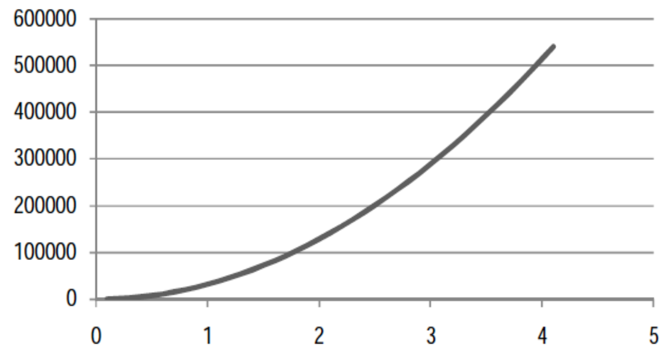

Figure 2: Cost versus precision when $f_1 = 0.5$.

4.2. Definite Number of Total Measurements

Assuming the total number of measurements is predetermined, the optimal fraction of direct measurements is determined at each level of budget between 76500 to 157500 SEK or each required precision from 1.478 to 3.297 for $N = 100$. Table **2** shows the cost needed and the precision yielded for each combined technique:

As shown, both cost and precision are increasing in f_1, while the cost of improving precision by one unit is decreasing in f_1. When a constraint is present either in budget or in precision, a combined technique is optimal. For instance, the maximum precision for a research budget that cannot exceed 100000 SEK is achieved with $f_1 = 0.25$, while the cost of achieving a precision of at least 2 is minimized by $f_1 = 0.6$. The values of MCBR show whether the necessary investment for improving the precision of an already implemented design may be funded.

5. DISCUSSION

The cost of achieving a required precision by using a combination of techniques to assess a working posture was calculated for different fractions of expensive direct technical measurements. Based on the empirical data on costs and variances associated with the two measurement methods, a certain level of precision (2.75) could be reached with around 346 indirect (observational) assessments at an estimated cost of 264967 SEK (cf. Table **1**). If the researcher began to add direct measurements in order to get accurate values, the cost would start to increase at a decreasing rate until the fraction reached 0.15. If,

Table 2: Calculation of Total Cost (TC), Precision of Combined Mean (P), and the Cost of Improving Precision by One Unit Compared to the First Alternative (MCBR). Costs are in SEK

N	f_1	$1-f_1$	n_1	n_2	TC	P	MCBR
100	0	1	0	100	76500	1.478	0
100	0.05	0.95	5	95	80550	1.508	133068
100	0.1	0.9	10	90	84600	1.540	125011
100	0.15	0.85	15	85	88650	1.575	117123
100	0.2	0.8	20	80	92700	1.612	109408
100	0.25	0.75	25	75	96750	1.652	101870
100	0.3	0.7	30	70	100800	1.695	94514
100	0.35	0.65	35	65	104850	1.741	87344
100	0.4	0.6	40	60	108900	1.791	80364
100	0.45	0.55	45	55	112950	1.846	73581
100	0.5	0.5	50	50	117000	1.907	67000
100	0.55	0.45	55	45	121050	1.974	60627
100	0.6	0.4	60	40	125100	2.048	54470
100	0.65	0.35	65	35	129150	2.132	48536
100	0.7	0.3	70	30	133200	2.226	42834
100	0.75	0.25	75	25	137250	2.334	37374
100	0.8	0.2	80	20	141300	2.460	32168
100	0.85	0.15	85	15	145350	2.609	27227
100	0.9	0.1	90	10	149400	2.789	22567
100	0.95	0.05	95	5	153450	3.011	18206
100	1	0	100	0	157500	3.297	14166

however, the researcher continued to increase the fraction of direct measurements, the cost of achieving the required precision would fall at an accelerating rate, and would reach its minimum when the fraction reaches unity (cf. Figure **1**). By using direct measurements in a combined technique, the researcher could improve the cost efficiency of the observation method since $\frac{\bar{c}_1}{\bar{c}_2} < \frac{\hat{\sigma}_2^2}{\hat{\sigma}_1^2}$, which does mean that the direct technical method offered much lower average cost of precision and then much higher cost efficiency compared to the observation method. If, however, the cost ratio $\bar{c}_1/\bar{c}_2$ would be larger than the variance ratio $\hat{\sigma}_2^2/\hat{\sigma}_1^2$ (i.e. the observation method offered lower average cost and thus higher cost efficiency compared to the direct method) the cost efficiency of the posture assessment study would increase by using more observational assessments in the combined technique and would maximize by using only the observation method. Thus, while a combined measurement technique (i.e. $0 < f_1 < 1$) could not be suggested for minimizing the cost of achieving the required precision, after a certain point the cost could definitely be reduced by increasing the fraction of direct measurements. However, when the total number of measurements was predetermined, the fraction could be optimized for a constraint either in budget or in precision (cf. Table **2**).

By using a regression technique, the cost of the statistical production could be estimated / predicted for any fraction of direct measurement at a constant level of precision, and also for any level of precision at a constant fraction.

5.1. Duality in Cost-Precision Association

Resolving the cost function (10) with respect to precision, we obtain the precision of the combined mean as a function of total cost and other parameters:

$$P = \left\{ \frac{TC}{\left[f_1 \bar{c}_1 + (1 - f_1) \bar{c}_2 \right] \cdot \left[f_1 \hat{\sigma}_1^2 + (1 - f_1) \hat{\sigma}_2^2 \right]} \right\}^{\frac{1}{2}} \quad (19)$$

Thus we can calculate the precision of the mean estimate for different combinations (i.e. different values of f_1) subject to a predetermined research budget. The results here would be the converse of those obtained when considering the situation with a predetermined precision: the budget-constrained precision would decrease as the fraction of direct measurements was increased from 0 to 0.15, but increase as the fraction was increased further from 0.15 to 1.00.

5.2. Diseconomies of Scale

The precision of the combined mean estimate expressed as equation (5) is increasing in measurement inputs n_1 and n_2 but at a decreasing rate; that is, the statistical production technology is characterized by decreasing returns to scale. This property results in a cost that grows exponentially with the precision, so that the marginal cost of precision is twice the average total cost. However, since the precision of the combined mean is not valued in the same terms as the cost, one should be careful with the conclusion that the statistical production technology exhibits diseconomies of scale. Thus, any combined measurement technique provided by determining the fraction of direct measurements would demonstrate decreasing returns to scale, which does also mean that the marginal cost is greater than the average total cost (cf. equations (11) and (12)).

5.3. Allocation in Long-Run Exposure Assessment Studies

The solution of the allocation problem requires enough information on the constant parameters, such as costs and variances associated with the two independent measurement methods. The variable cost for each technical method usually consists of all costs that vary with the number of measurements. In long-run data production, no fixed costs are considered because all costs vary. Distinguishing between long-run and short-run data production in exposure assessment studies is therefore very important when attempting to evaluate alternative measurement methods economically. The basic assumption in optimizing the combined technique was that the exposure assessment study was carried out in the long-run case; that is, where both types of measurements were allowed to vary. However, in a short-run exposure assessment study, where one type of measurement is fixed, the derived cost function cannot be used to find the appropriate combined technique.

In short-run studies, when fixed costs are of major importance in any economic decision, a comparison analysis of alternative measurement methods could be used to identify the measurement method that is relatively most cost-efficient [13, 18, 19], or the cost function could be rearranged in order to determine the variable input in terms of the fixed input. Thus, if the researcher is only allowed to adjust one of the measurement methods (short-run decision), the demand function will be dependent on the amount of the fixed input in addition to the input prices and the precision.

5.4. Model for Estimating Total Variable Cost

As pointed out above, the allocation problem was reasonably resolved in the case of a long-run exposure assessment study; and thus the isocost line equation (6) consisted of stable average variable costs with linear characteristics. When the average costs are unstable due to a typically fast labour productivity growth at the beginning of a statistical production, the average costs develop non-linearly with the number of measurements/estimates. In the long run, however, the average cost will not develop non-linearly, as the labour productivity growth in exposure measurements usually smoothes out and becomes stable. The long run cost minimization problem should thus be resolved by using the equation (6) and the stable average unit costs of a direct measurement and an indirect estimate.

5.5. The Impact of Systematic Errors

In this study, the ability of the combined measurement technique to produce information on exposure was evaluated by using the combined mean variance (3). However, systematic errors (bias) produced by the measurement methods, which reduce this ability, would influence the results. Reducing the measurement bias associated with an indirect subjective method and/or the selection bias associated with direct technical methods can be a reason in itself to use a combined technique [23]. However, the applied error equation, and thus the derived functions, did not account for any 'constant' error associated with the composited inputs. To get the cost function to include systematic errors, the mean square error of the combined mean, $MSE(\hat{\mu}_c)$, containing the systematic errors produced by the techniques (B_1^2 and B_2^2) can replace the applied variance formula (3) in the cost-efficiency analysis.

5.6. Minimizing the Cost of a Combined Technique for an Indefinite N: A Recommendation

Differentiating the cost function (10) with respect to f_1, and equating this to zero, the optimized fraction of direct measurements is obtained as follows:

$$f_1^* = \frac{\bar{c}_1\hat{\sigma}_2^2 - \bar{c}_2\left(2\hat{\sigma}_2^2 - \hat{\sigma}_1^2\right)}{2\left(\bar{c}_1 - \bar{c}_2\right)\left(\hat{\sigma}_2^2 - \hat{\sigma}_1^2\right)}, \tag{20}$$

provided the following conditions to satisfy $0 < f_1^* < 1$ are fulfilled for $2\hat{\sigma}_1^2 > \hat{\sigma}_2^2$:

$$\frac{2\hat{\sigma}_2^2 - \hat{\sigma}_1^2}{\hat{\sigma}_2^2} < \frac{\bar{c}_1}{\bar{c}_2} < \frac{\hat{\sigma}_1^2}{2\hat{\sigma}_1^2 - \hat{\sigma}_2^2} \tag{21}$$

By substituting (20) into the cost function (10) to minimize the cost of achieving a required precision by using a combined technology, the following optimal (value) function (cost function) is obtained:

$$C^* = \frac{P^2 \cdot \left(\bar{c}_1\hat{\sigma}_2^2 - \bar{c}_2\hat{\sigma}_1^2\right)^2}{4\left(\hat{\sigma}_2^2 - \hat{\sigma}_1^2\right)\left(\bar{c}_1 - \bar{c}_2\right)} \tag{22}$$

However, a (minimized) cost function has to be concave in *c* – that is, $\partial^2 C(c,P)/\partial c_i^2 \le 0$ must hold – which does not apply to the function (22). It is worth noting that the underlying precision formula does not provide a quasi-convex *isoquant curve*, which contains all combinations of direct and indirect measurements giving the same precision. Thus, the isoquant curve has no minimizing point. The cost of the combined assessment technique can, however, be minimized when an appropriate function is used to estimate the *value of information* (VOI) produced by direct measurements and indirect estimates. The function should provide a quasi-convex isoquant. The VOI is the amount of money a decision maker would be willing to pay for information prior to making an economic decision. Regarding exposure assessment studies, the statistical efficiency (precision and/or accuracy) is only one factor determining the amount of money a decision maker would be willing to pay; the value of information produced is also determined by the *usefulness* of the information in further research and the expected social *benefit* of the exposure assessment study. Thus, we recommend that the output of the statistical production (i.e. the value of information produced by the combined technique) is estimated by using the following CES production function:

$$Y = A \cdot \left[\delta \cdot n_1^{-\rho} + \left(1-\delta\right) \cdot n_2^{-\rho}\right]^{-\frac{1}{\rho}} \tag{23}$$

Here, $A > 0$ exhibits the technological effect on the output *Y* (in this case, the productivity of the combined technique); n_1 and n_2 are the numbers of direct and indirect measurements, respectively; the distribution parameter being bounded between zero and unity, $0 \le \delta \le 1$, reflects the intensity of direct technique in the statistical production; and $\rho \ge -1$ is a constant reflecting the output elasticities of inputs.

The Lagrangian for resolving the cost minimization problem of the combined technique is:

$$L\left(n_1, n_2, \lambda\right) = n_1\bar{c}_1 + n_2\bar{c}_2 \\ -\lambda\left(\left[\delta \cdot n_1^{-\rho} + \left(1-\delta\right) \cdot n_2^{-\rho}\right]^{-\frac{1}{\rho}} - Y^0\right), \tag{24}$$

where $\bar{c}_1$ and $\bar{c}_2$ are the average unit costs of a direct and an indirect measurement, respectively, and λ is the Lagrange multiplier.

The first-order conditions lead to the optimization condition as follows:

$$\frac{\bar{c}_1}{\bar{c}_2} = \frac{\delta n_1^{-(1+\rho)}}{\left(1-\delta\right) n_2^{-(1+\rho)}}, \tag{25}$$

which can be rearranged as the following logarithmic regression equation to estimate the *elasticity of substitution* $\left(\sigma = \dfrac{1}{1+\rho}\right)$ between n_1 and n_2:

$$\ln\left(\frac{n_1}{n_2}\right) = a + \sigma \cdot \ln\left(\frac{\bar{c}_2}{\bar{c}_1}\right) \tag{26}$$

The optimization condition is used to derive the conditional demand functions for n_1 and n_2, and by substituting the demand functions into the isocost equation, we obtain the optimal (value) function (cost function) which relates the minimized cost to $\bar{c}_1$ and $\bar{c}_2$, as well as to the required value of information produced:

$$C^{\min} = K \cdot \left(a\bar{c}_1^{\,k} + b\bar{c}_2^{\,k}\right)^{\frac{1}{k}} \cdot Y^{\frac{1}{\mu}}, \tag{27}$$

where *C* is the total cost; *K* is a function of *A*; *a* and *b* are function of *δ*; *k* is a function of *ρ*; and *μ* is the returns to scale. Hence the cost function above is concave in *c*.

6. CONCLUSION

When combining an expensive direct technical method with an inexpensive subjective method in an exposure assessment study, increasing the fraction of expensive direct measurements can, unexpectedly, reduce the cost of achieving a desired level of precision. Thus, the average cost of an exposure measurement is not the determining factor in cost efficiency analysis; the decisive factor is the average and marginal cost of precision. In principle, if a SEK spent on direct measurements is more productive than a SEK spent on indirect estimates, the decision-maker will want to use more direct measurements.

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