# Circular Statistical Analysis of Emergency Department Admissions

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**Abstract:** Health-care administrators face ongoing challenges managing emergency department (ED) operations, particularly in understanding how patient arrival trends fluctuate within the 24-hour day. Although prior research has examined the times at which patients seek emergency care, most of these studies have used simple statistical methods that do not account for time as a periodic variable. As a result, many significant time-of-day patterns may not be detected. We use circular statistics on 142,005 hourly emergency department admissions at a large hospital in Iowa from January 2014 to August 2017. Overall, the pattern of ED visits presents an anisotropic distribution that is statistically significant according to both Rayleigh and Kuiper tests. Patient arrival times show a circular mean in the early to mid-afternoon, a marked late-afternoon modal peak, and a diffuse distribution across the day. Adjusted circular probability models such as the von Mises distribution, cardioid distribution, and the wrapped normal distribution perform significantly better than the circular uniform model when the AIC, BIC, CAIC, and HQIC criteria are considered. The circular summary charts help in understanding the various trends observed in the time series graph. In pointing out how the use of a circular method is a mathematically appropriate and more interpretable approach for describing trends related to admissions on an hourly basis, this piece of research also points out the benefits of such a method as being a useful tool for health-care planners.

**Keywords:** Circular Statistics, Emergency Department, Temporal Patterns, von Mises Distribution, Uncertainty Quantification, Healthcare Operations.

## 1. INTRODUCTION

Emergency departments (EDs) serve as vital gateways for providing emergency medical services, a setting for immediate and unplanned treatment. Increasing demand for ED services has placed further pressure on staffing, patient throughput, and crowd management across the world. One of the main questions that underlies these issues is: *when do patients tend to arrive, in general?*

Existing work consistently shows temporal variation in ED and emergency service utilization. Studies have documented pronounced daily, weekly, and seasonal patterns in ED arrivals and length of stay, with peaks at particular hours of the day and substantial variation across days and months [1-4]. Forecasting studies have employed regression, classical time-series models, and machine learning to predict ED arrivals or occupancy [5-9], informing staffing and bed-management decisions.

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However, most prior investigations treat time-of-day using linear methods that do not fully respect the cyclical structure of the 24-hour clock. A clock reading 23:00 and one reading 01:00 are only two hours apart on the circle, but appear far apart on a linear scale. Classical treatments of circular data [10-13] and more recent practical guidance [14-17] stress that hours, directions, and phases are inherently periodic and require specialized methods. Circular statistics provide tools such as circular means and variances, circular uniformity tests (e.g., Rayleigh and Kuiper tests), and circular probability distributions (e.g., von Mises and wrapped normal), which are now widely used in biology, environmental science, and the social sciences [15,18].

Despite a growing body of work on ED forecasting [5-9] and temporal patterns in arrivals [1,3], the explicit use of circular statistics for hour-of-day ED arrivals remains uncommon. Related health research has illustrated how sine-cosine terms can capture periodic seasonal patterns [19], but this approach is rarely extended to the 24-hour cycle via circular methods. Recent work has demonstrated the value of circular statistics in medical contexts such as analyzing circadian patterns in blood pressure [20] and activity rhythms in critical care settings [21], yet their application to ED operational planning remains underexplored.

To address this methodological gap, we conduct a comprehensive circular statistical analysis of hourly ED admission data covering January 2014 through August 2017 from a major tertiary hospital in Iowa. Unlike previous ED forecasting studies, where time is treated linearly or represented using ad-hoc hour-of-day indicators, our work explicitly applies the full machinery of circular statistics–including circular summary measures, formal uniformity tests, and parametric circular distribution fitting–to characterize the 24-hour rhythm in ED arrivals. Specifically, this study: (1) demonstrates that circular methods yield more mathematically appropriate and interpretable characterizations of hourly ED patterns than linear approaches; (2) provides a rigorous statistical framework for identifying and quantifying daily admission rhythms using established circular distributions; (3) presents useful visualizations, including rose diagrams and circular summaries, that can directly inform operational decision-making; and (4) establishes a methodological template for applying circular statistics to other cyclical healthcare processes.

The dataset was compiled by Choudhury and Urena and is publicly available in the Dryad repository (https://datadryad.org/dataset/doi:10.5061/dryad.q57d4g4) [5]. It has been used previously for hourly ED forecasting based on traditional time-series models. By applying circular measures of central tendency, dispersion, distributional fitting, and formal uniformity tests, this work offers a more accurate representation of the daily rhythm of emergency care utilization and complements existing forecasting studies.

## 2. MATERIALS AND METHODS

### 2.1. Study Design, Setting, and Dataset

We performed a retrospective observational study using de-identified hourly ED admission data from a large tertiary academic hospital in Iowa. The dataset, compiled by Choudhury and Urena and made publicly available in the Dryad repository (https://datadryad.org/dataset/doi:10.5061/dryad.q57d4g4) [5], contains hourly arrival counts aggregated over the period January 2014 to August 2017. For each hour of the day (00:00 through 23:00), the dataset reports the total number of ED admissions occurring in that hour over the full study period. All personal identifiers were removed prior to analysis, and only aggregated counts were used.

### 2.2. Data Preparation, Descriptive Analysis, and Circular Transformation

Each of the 24 hours was treated as a discrete point on the 24-hour clock. Let $h_i \in \{0,1,\ldots,23\}$ denote the hour index. To apply circular statistical methods appropriately [10,11,14], we converted these clock hours into angular measurements in radians using

$$\theta_i = \frac{2\pi h_i}{24}, \quad i = 0,\ldots,23, \tag{1}$$

mapping the 24-hour day onto the unit circle, with midnight (00:00) at angle $0$, noon (12:00) at $\pi$, and so on in steps of $\frac{2\pi}{24}$.

Let $n_i$ denote the total number of admissions recorded at hour $h_i$ across the study period. To account for these differing frequencies, we formed a frequency-weighted angular sample by repeating each $\theta_i$ exactly $n_i$ times. This expanded vector of angles represents each individual admission as a point on the circle and is the basis for all circular computations, as recommended for grouped circular data [15-17].

While this frequency-weighted expansion is the standard approach for grouped circular data and allows proper application of circular statistical methods, it does have limitations. Most notably, by aggregating all admissions within each hour into a single angular value, we lose information about within-hour variability in arrival times. For instance, admissions occurring at 14:05 and 14:55 are both treated identically as occurring at hour 14. This aggregation may smooth over finer temporal patterns such as quarter-hour or half-hour cycles. Additionally, the method assumes that each admission within an hour contributes equally to the circular pattern, which may not fully capture variations in patient acuity or service demand throughout the hour. Despite these limitations, hourly aggregation remains the most common temporal resolution in ED operational data [1,2,5], and our approach faithfully represents the information available in such aggregated datasets. Future work with finer-grained timestamps could address within-hour variability more directly.

In parallel, we produced simple descriptive summaries in the original hour scale: total counts per hour, a line plot of hourly admissions across 00:00–

23:00, and a heatmap of hourly counts with wrap-around from 23:00 to 00:00. These linear visualizations provide an intuitive first impression of the data, while the circular diagrams emphasize the cyclical nature of time-of-day.

### 2.3. Circular Statistical Methods and Distribution Fitting

We computed classical circular summary statistics as described by Fisher [10] and Mardia and Jupp [11]. Given an expanded angular sample $\{\theta_j\}_{j=1}^{n}$, we first formed the mean sine and cosine components:

$$\bar{C} = \frac{1}{n}\sum_{j=1}^{n}\cos\theta_j, \tag{2}$$

$$\bar{S} = \frac{1}{n}\sum_{j=1}^{n}\sin\theta_j. \tag{3}$$

The mean resultant length $R$ is

$$R = \sqrt{\bar{C}^2 + \bar{S}^2}, \tag{4}$$

with $R \in [0,1]$. Intuitively, $R$ measures how tightly the data cluster around a single direction: $R = 1$ indicates perfect concentration at one point on the circle, while $R = 0$ indicates complete dispersion uniformly around the circle. The circular mean direction $\bar{\theta}$ is given by

$$\bar{\theta} = atan2(\bar{S}, \bar{C}), \tag{5}$$

mapped to $[0, 2\pi)$, and was subsequently converted back to hours by

$$\bar{h} = \frac{24}{2\pi}\bar{\theta}. \tag{6}$$

The circular mean represents the “typical” or “average” time of day at which admissions occur, accounting for the circular nature of the clock.

The circular variance was defined as

$$V = 1 - R, \tag{7}$$

and the circular standard deviation $S$ was computed using the standard approximation [10,11]

$$S = \sqrt{-2\ln R}, \tag{8}$$

expressed both in radians and in hours by scaling $S$ by $\frac{24}{2\pi}$. A larger circular standard deviation indicates greater spread of admissions throughout the day, while a smaller value indicates tighter clustering around the mean direction.

In addition to these measures, we identified the circular median (defined as the 50th percentile of the angular sample) and circular mode (the hour with maximum admission count).

We then evaluated four circular probability models commonly recommended for circular data [11,14,15]: the circular uniform distribution, the von Mises distribution, the cardioid distribution, and the wrapped normal distribution. These distributions are all unimodal and were selected based on standard practice in circular data analysis [10,14]. Here we restricted attention to unimodal circular distributions because the empirical data exhibit a single clear peak in the late afternoon (hour 17), with no secondary peaks of comparable magnitude (Figure **2**). While multimodal circular distributions such as mixtures of von Mises distributions exist [11,13], their use is typically justified when multiple distinct peaks of similar prominence are present—for instance, in biological contexts with bimodal activity rhythms or navigational data with multiple preferred directions. In our dataset, admissions decline smoothly from the late-afternoon peak through the evening and night, without a second distinct mode. Introducing additional parameters for multimodal models without empirical justification would risk overfitting and complicate interpretation. Nonetheless, if future studies reveal secondary peaks (e.g., a minor morning surge in certain EDs), mixture models or other multimodal approaches [13] would be appropriate extensions.

#### *2.3.1. Uniform Circular Distribution*

The circular uniform model assumes all directions are equally likely, with density

$$f_U(\theta) = \frac{1}{2\pi}, \quad 0 \le \theta < 2\pi. \tag{9}$$

#### *2.3.2. Von Mises Distribution*

The von Mises distribution has density

$$f_{VM}(\theta;\mu,\kappa) = \frac{1}{2\pi I_0(\kappa)}\exp\{\kappa\cos(\theta-\mu)\}, \tag{10}$$

where $\mu$ is the mean direction, $\kappa \ge 0$ is the concentration parameter, and $I_0(\kappa)$ is the modified Bessel function of the first kind and order zero [10,11]. *Larger values of $\kappa$ indicate stronger clustering of data around the mean direction $\mu$, analogous to smaller variance in linear statistics. When $\kappa = 0$, the von Mises distribution reduces to the circular uniform distribution.*

#### 2.3.3. Cardioid Distribution

The cardioid distribution is a simple unimodal circular distribution with density

$$f_C(\theta;\mu,\rho) = \frac{1+2\rho\cos(\theta-\mu)}{2\pi}, \tag{11}$$

where $\mu$ is the mean direction and $\rho$ is a concentration parameter satisfying $0 \le \rho \le 0.5$ to ensure a valid density [11]. *Larger $\rho$ values correspond to stronger clustering around $\mu$, with $\rho = 0$ yielding the uniform distribution and $\rho = 0.5$ producing maximum asymmetry.*

#### 2.3.4. Wrapped Normal Distribution

The wrapped normal distribution wraps a normal variate with mean $\mu$ and standard deviation $\sigma$ onto the unit circle [11,13]. Its density at angle $\theta$ is

$$f_{WN}(\theta;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \sum_{k=-\infty}^{\infty} \exp\left\{-\frac{(\theta-\mu+2\pi k)^2}{2\sigma^2}\right\}, \tag{12}$$

with the sum truncated numerically to a small range of $k$ values in practice. The parameter $\sigma$ controls the degree of wrapping and concentration: smaller $\sigma$ yields tighter clustering, while larger $\sigma$ corresponds to greater dispersion.

For the von Mises, cardioid, and wrapped normal models, parameters $(\mu,\kappa)$, $(\mu,\rho)$, and $(\mu,\sigma)$ were estimated by maximum likelihood using the expanded angular sample, following standard implementations [10,14,15]. The uniform model has no free parameters.

To compare the fit of these distributions, we used the log-likelihood $\ell$ and four information criteria: Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), corrected AIC (CAIC), and the Hannan-Quinn Information Criterion (HQIC). For each model with $k$ free parameters and sample size $n$, these are defined as

$$AIC = 2k - 2\ell, \tag{13}$$

$$BIC = k\ln(n) - 2\ell, \tag{14}$$

$$CAIC = AIC + \frac{2k(k+1)}{n-k-1}, \tag{15}$$

$$HQIC = 2k\ln(\ln n) - 2\ell, \tag{16}$$

as commonly used in ED forecasting and model-comparison studies [5-9].

### 2.4 Tests for Uniformity and Goodness-of-Fit

To assess whether arrivals were uniformly distributed around the circle, we applied two widely used circular uniformity tests [10,11,17]: the Rayleigh test and Kuiper test.

The Rayleigh test statistic is defined as

$$Z = nR^2, \tag{17}$$

which, under the null hypothesis of circular uniformity, has an approximate distribution that allows computation of a $p$-value [10]. This test is particularly powerful for unimodal, non-uniform alternatives.

The Kuiper test compares the empirical cumulative distribution function (ECDF) of the angles with the CDF of the circular uniform distribution and is sensitive to deviations anywhere on the circle [11,17]. It is a circular analogue of the Kolmogorov-Smirnov test and is invariant under rotation of the circle, making it suitable for detecting both localized and global deviations from uniformity.

In addition to these formal tests, we visually compared the empirical histogram of hourly admissions with the fitted circular probability density functions (PDFs), and we compared the ECDF with fitted circular cumulative distribution functions (CDFs) for the uniform, von Mises, cardioid, and wrapped normal models [15,17].

### 2.5. Software

All analyses were performed in Python (version 3.11). Numerical computation and data handling were implemented using NumPy and Pandas, circular statistics and distribution fitting using SciPy, and plotting using Matplotlib. This combination follows recent recommendations for reproducible circular data analysis and visualization [15,17,18].

## 3. RESULTS AND DISCUSSION

### 3.1. Circular Statistical Summary

Table **1** presents the circular statistical measures for hourly ED admissions. The circular mean of 15.3 hours corresponds to approximately 15:15 (3:15 PM), identifying this period as the typical admission hour. The circular median of 14.0 hours (14:00, 2:00 PM) yields a similar estimate, confirming that arrivals tend to cluster in the early-to-mid-afternoon range. The circular mode at 17 hours (17:00, 5:00 PM) shows that this single hour recorded the highest frequency of admissions. The mean resultant length $R = 0.27$ indicates weak overall clustering when compared to standard benchmarks [10, 11], which suggests the presence of an afternoon peak while admissions remain fairly distributed across the 24-hour cycle. In specific terms, an $R$ value of 0.27 indicates a dominant daily rhythm that is not tightly concentrated at a single time—ED demand remains substantial throughout many hours of the day rather than spiking sharply in one narrow window. The corresponding circular variance $V = 0.73$ and circular standard deviation $S = 1.62$ radians (approximately 6.2 hours) reinforce this impression of a broad spread. The fitted von Mises distribution resulted in a mean direction parameter $\mu = 15.3$ hours (15:15), closely matching the circular mean and confirming that the parametric model places the peak in the same afternoon window. The concentration parameter $\kappa = 0.56$ represents modest clustering, in agreement with the observed mean resultant length.

Rayleigh's test statistic was $Z = 10174.63$ with a $p$-value below 0.001, whereas the Kuiper test statistic was $V = 0.22$, also with $p < 0.001$. Together, these tests give strong evidence against the null hypothesis of a uniform 24-hour arrival pattern, confirming a pronounced daily rhythm in ED admissions [10, 16]. These fluctuations pose an operational challenge, as the results show that ED arrivals are far from random within the course of a day. The very high degree of significance of non-uniformity points to predictable, systematic variation in demand, which can be leveraged for operational planning. Specifically, the concentration of arrivals in the afternoon and early evening suggests that staffing and resource allocation should be adjusted accordingly, rather than being spread uniformly across all hours. The shape of this rhythm—low activity overnight, rising through the morning, peaking from early afternoon into early evening, and declining again at night—is consistent with prior descriptions of ED arrival patterns and crowding dynamics [1-3].

The above summaries demonstrate that the ED displays a strong circadian structure, with arrivals concentrated from early afternoon to early evening but non-negligible volumes across much of the day. Such patterns align with previous observational work documenting daytime peaks and nighttime troughs [1, 3] and complement studies on time-series forecasting that consider the hour of day as an important predictor [2, 5, 6].

### 3.2. Visualization of Circular Patterns

Figure **1** above shows the rose diagram (circular histogram) of hourly ED admissions with the circular mean direction overlaid. The length of the bars represents the frequency of admissions in that hour, plotted at its corresponding angle, making the cyclical nature of the data visually explicit. The direction arrow indicates that, on average, patients entered the ED in the afternoon, prior to the peak hour.

The line plot in Figure **2** depicts ED admissions on a linear time-of-day axis, with vertical lines indicating the circular mean, circular median, and mode. This view complements the rose diagram by presenting the same information in a familiar format while emphasizing the afternoon and early-evening peak.

Figure **3** summarizes mean direction and dispersion on the circle. The mean resultant vector, of length $V = 0.22$, points to the estimated mean arrival time,

**Table 1: Circular Statistical Measures of Hourly ED Admissions**

| Statistic | Value | Interpretation |
|---|---|---|
| Circular Mean | 15.3 hours | Typical admission hour |
| Circular Median | 14.0 hours | Middle point |
| Circular Mode | 17 hours | Highest count |
| Mean Resultant ( $R$ ) | 0.27 | Weak clustering |
| Circular Variance ( $V$ ) | 0.73 | High variance |
| Circular Std Dev ( $S$ ) | 1.62 rad | Dispersion |
| Circular Std Dev | $\approx 6.2$ hrs | Dispersion in hours |
| VM Mean ( $\mu$ ) | 15.3 hours | Peak time |
| VM Concentration ( $\kappa$ ) | 0.56 | Concentration |
| Rayleigh Stat ( $Z$ ) | 10174.63 | Non-uniform |
| Rayleigh $p$-value | $< 0.001$ | Significant |
| Kuiper Stat ( $V$ ) | 0.22 | Non-uniform |
| Kuiper $p$-value | $< 0.001$ | Significant |

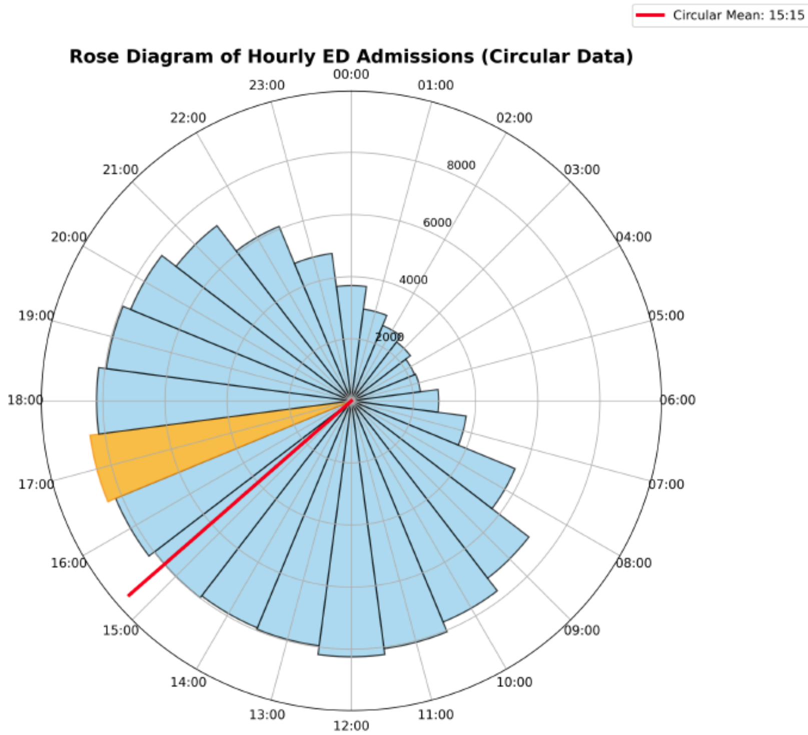


**Figure 1:** Rose diagram of hourly ED admissions with circular mean direction indicated. Bars represent total admissions per hour, mapped to angles on the 24-hour clock.

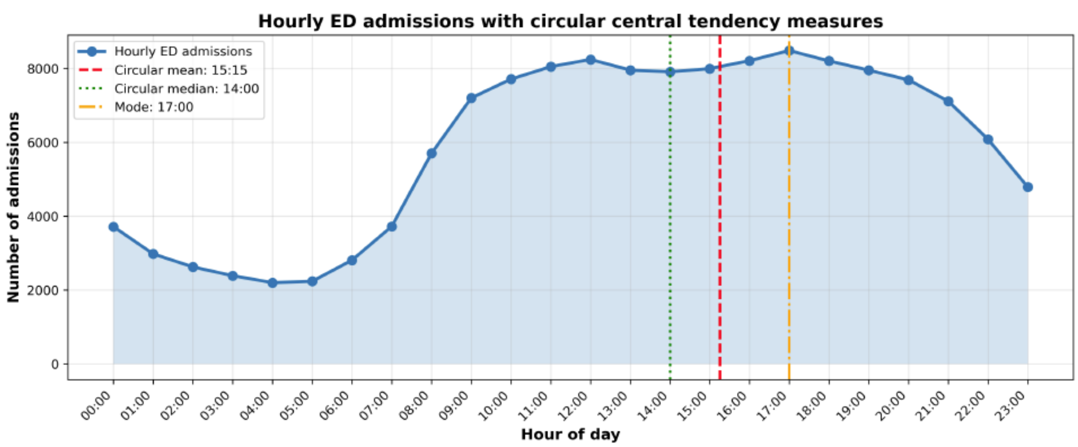


**Figure 2:** Hourly ED admissions across the 24-hour day, with circular mean, median, and mode indicated as vertical reference lines.

while the shaded band indicates approximately $\pm 1$ circular standard deviation. As recommended for circular data visualization [15,17], this plot communicates both the dominant direction and the degree of spread in a way that is directly interpretable on the original circular scale.

### 3.3 Distributional Modelling and Information-Criteria Comparison

To assess how well different circular probability models represent the observed pattern, we fitted the circular uniform, von Mises, cardioid, and wrapped normal distributions to the expanded angular sample. Figure **4** shows the empirical histogram of hourly angles with the fitted circular PDFs overlaid. The continuous curves are scaled so that their total area matches the total number of admissions, allowing direct comparison between observed counts and model-based expectations.

The fitted von Mises, cardioid, and wrapped normal curves track the empirical shape of the histogram much more closely than the flat uniform curve, particularly in capturing the afternoon peak and the extended

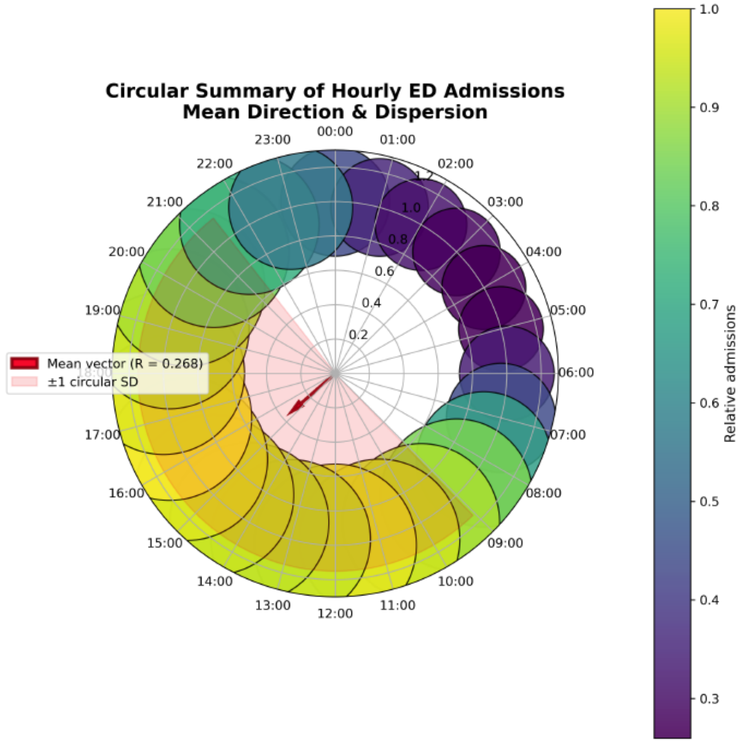


**Figure 3:** Circular summary of mean direction and dispersion. The arrow shows the mean resultant vector; the shaded region indicates $\pm 1$ circular standard deviation around the mean direction.

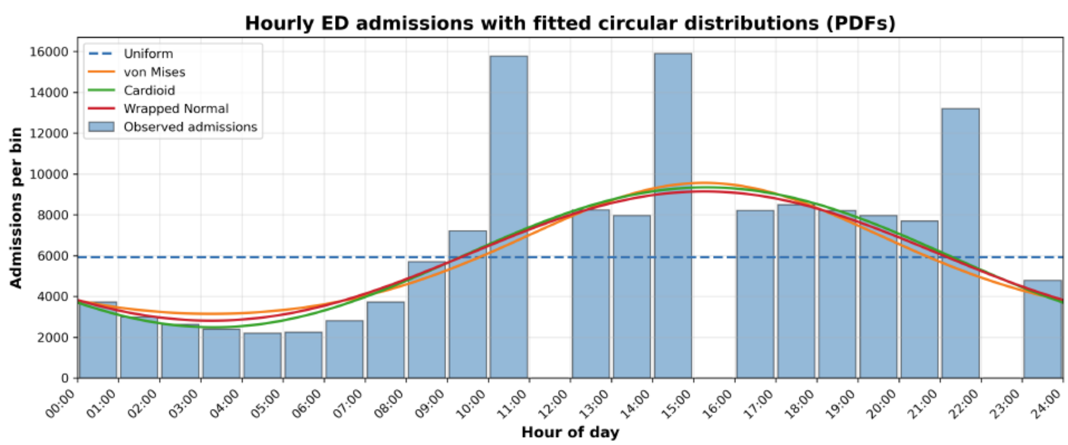


**Figure 4:** Histogram of hourly ED admission angles with fitted circular probability density functions: uniform, von Mises, cardioid, and wrapped normal. Densities are scaled to histogram counts to aid visual comparison.

shoulder into early evening. This visual impression is corroborated by the CDF comparison in Figure **5**, where the empirical CDF is plotted against the fitted uniform, von Mises, cardioid, and wrapped normal CDFs. All three non-uniform models follow the empirical CDF closely across the entire circle, whereas the uniform CDF deviates substantially.

To quantify relative fit quality, Table **2** summarizes the information-criteria comparison across the four models, reporting AIC, BIC, CAIC, and HQIC. Lower values indicate better fit, with penalties for model complexity. The cardioid model attains the lowest AIC, BIC, CAIC, and HQIC values, closely followed by the wrapped normal model; both substantially outperform the circular uniform model. The von Mises model also improves markedly on the uniform benchmark, indicating that the additional parameters in these non-uniform models are justified by improved fit to the data [11,14,5-9].

While the cardioid model achieves the lowest information criteria values, the wrapped normal model performs nearly as well, with AIC and BIC differences

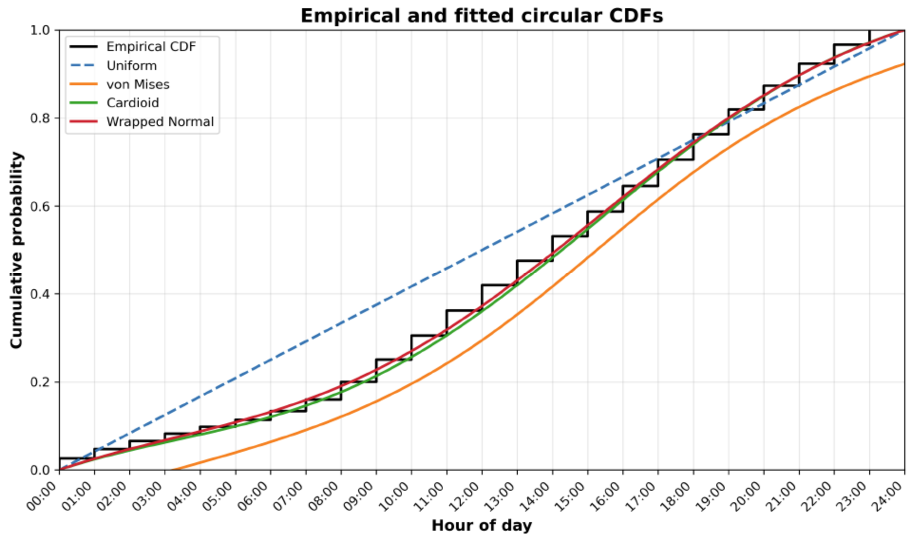


**Figure 5:** Empirical cumulative distribution function (ECDF) of ED arrival angles and fitted circular CDFs for the uniform, von Mises, cardioid, and wrapped normal models.

**Table 2: Model Comparison Based on Information Criteria for the Cardioid, von Mises, Wrapped Normal, and Circular Uniform Models. Lower Values Indicate Better Fit**

| Distribution | $k$ | LogLik | AIC | BIC | CAIC | HQIC |
|---|---|---|---|---|---|---|
| Wrapped Normal | 2 | -249667.385 | 499338.771 | 499358.498 | 499338.771 | 499344.665 |
| Cardioid | 2 | -249458.215 | 498920.430 | 498940.158 | 498920.431 | 498926.324 |
| von Mises | 2 | -250623.207 | 501250.415 | 501270.142 | 501250.415 | 501256.308 |
| Uniform | 0 | -260987.733 | 521975.466 | 521975.466 | 521975.466 | 521975.466 |

of only ∶400–420 units on a scale exceeding 498,000. In practical terms, both models capture the empirical distribution very effectively, and the small numerical advantage of the cardioid may have limited operational relevance. The von Mises model, though slightly less optimal statistically, still fits the data far better than the uniform model and remains a popular choice due to its mathematical tractability and widespread use in circular statistics [10,11]. For ED operational planning, any of these three non-uniform models would provide a reasonable basis for characterizing the daily rhythm. The key practical insight is not which non-uniform model is marginally superior, but rather that *all* non-uniform models vastly outperform the uniform assumption, underscoring the necessity of accounting for circadian structure in ED demand modeling. The choice among cardioid, wrapped normal, and von Mises distributions may thus be guided by computational convenience, interpretability, or integration with existing forecasting frameworks, rather than by small differences in information criteria [14,15].

These findings align with recent forecasting work that emphasizes the importance of flexible, non-uniform models in capturing ED arrival patterns [5-9]. While our focus is on the static 24-hour pattern rather than multihorizon forecasting, the same principle applies: models that explicitly encode diurnal structure yield more realistic representations of ED demand.

### 3.4. Heatmap and Circadian Structure

Figure **6** presents a heatmap of hourly ED admissions across the circular day, with the 24th column repeating 00:00 to show wrap-around continuity. Darker cells indicate higher admission counts. This visualization highlights the sustained band of elevated activity from early afternoon into the evening and the relative quiet during late-night and early-morning hours.

Heatmap-style presentations of temporal variation have been advocated as intuitive tools for communicating ED crowding patterns to clinicians and managers [1,8,9]. In our context, combining the heatmap with circular plots reinforces the conclusion that ED arrivals follow a pronounced circadian structure that should be reflected in staffing and operational planning.

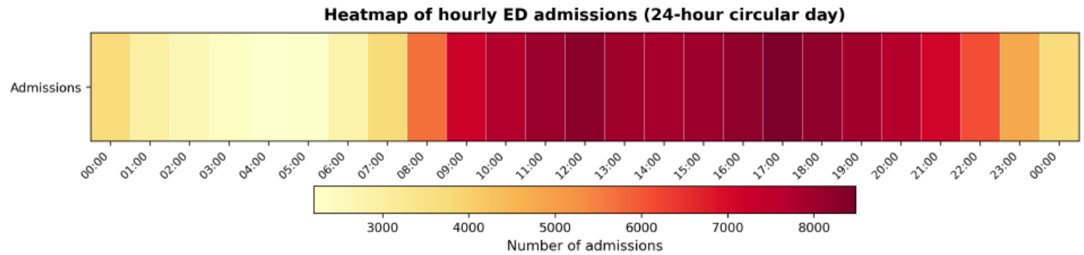


**Figure 6:** Heatmap of hourly ED admissions with circular wrap-around, emphasizing the afternoon and early-evening peak and reduced nighttime arrivals.

### 3.5 Relation to Previous Work and Practical Implications

The daily pattern uncovered by circular statistics in this dataset is consistent with prior observational and forecasting studies of ED arrivals. For example, Hertzum [1] reported strong effects of hour-of-day on ED patient volume in Denmark, with daytime peaks and nighttime troughs. Tiwari *et al*. [3] documented similar temporal concentration in an Indian tertiary hospital, and Jones *et al*. [2] showed that hour-of-day and related variables substantially improve forecasting performance.

More recent ED forecasting research has investigated advanced time-series, regression, and machine-learning approaches [5-9], often combining calendar and meteorological features. Our analysis supplements these efforts by focusing on the inherent circular character of hour-of-day. Circular approaches provide interpretable metrics such as the mean direction, the mean resultant length, and the circular variance, which are important characteristics of circular distributions. These quantities can be directly communicated to hospital leadership as â€œtypical peak timeâ€← and â€œdegree of concentration around the peakâ€← rather than abstract regression coefficients.

To illustrate the practical utility of these findings, consider a hypothetical ED currently using uniform 8-hour staffing shifts (e.g., shifts starting at 00:00, 08:00, and 16:00, each with equal staffing levels). Our analysis shows that admissions peak around 17:00 (5:00 PM) and remain elevated from roughly 12:00 to 22:00, with a circular mean at 15:15 and substantial dispersion (± 6.2 hours). Based on these findings, ED leadership could:

**Increase staffing during the peak window (14:00–20:00):** Assign additional nurses, physicians, and support staff to the afternoon and early-evening shift to handle the documented surge in arrivals. For instance, if the baseline staffing ratio is 1 physician per 10 patients per hour, the peak hours (15:00–18:00) might justify increasing to 1 physician per 8 patients to maintain service quality.

**Reduce staffing during the trough (02:00–06:00):** With admissions reduced to approximately 2,200–3,000 per hour during early morning hours (as opposed to 8,485 at the peak), maintaining full daytime staffing levels overnight is inefficient. Shifting one or two staff members from the overnight shift to the afternoon shift could improve resource utilization without compromising the quality of care during low-demand hours.

**Implement flexible shift start times:** Instead of rigid 8-hour blocks, introduce staggered shifts starting at 13:00, 14:00, and 15:00 to fit workforce availability to the growing demand curve revealed by Figure **2**.

**Pre-position resources for the afternoon surge:** Ensure that examination rooms, diagnostic equipment, and ancillary services are fully prepared by 14:00 each day to accommodate the predictable increase in patient flow.

Such alterations, grounded in the quantified circular pattern, can reduce wait times during peak hours, improve staff satisfaction by better matching workload to staffing levels, and enhance overall ED efficiency [1, 9]. The circular mean and the parameters of dispersion provide clear and interpretable targets for such operational decisions, translating statistical findings into staffing policies in a straightforward fashion.

From an operational viewpoint, the above-identified afternoon to evening peak supports the concentration of staff and resources in this window, while recognizing that substantial arrivals continue across much of the day. This is in keeping with other literature on best practices for aligning ED staffing according to demand to prevent crowding.

### 3.6. Study Limitations

Several limitations may be noted for these results. First, our analysis rests on data from a single hospital in Iowa over a 3.5-year period, which may limit generalizability to other geographic regions, healthcare systems, or time periods. Different EDs may have distinct temporal patterns due to local demographics, service availability (e.g., proximity to urgent care clinics), and cultural factors affecting healthcare-seeking behavior [3, 4].

Second, the dataset aggregates admissions across multiple years (January 2014 to August 2017), which obscures year-to-year variation in utilization or long-term trend data. As a result, potential changes in arrival patterns over time are not directly visible in the aggregated visualizations. Seasonal effects, day-of-week patterns, and holiday influences are not captured by our hourly analysis. While circular methods are well-suited to the 24-hour cycle, they do not address these longer periodicities, which may also be operationally relevant [1,2].

Third, we analyzed only aggregated hourly counts without access to patient-level covariates such as age, sex, chief complaint, triage acuity, or service times. Consequently, we cannot determine whether the observed circadian pattern is uniform across patient subgroups or whether certain types of cases (e.g., trauma vs. medical complaints) exhibit different temporal rhythms. Future work incorporating patient-level data could refine our understanding of how different populations contribute to the overall diurnal pattern [9]. Fourth, as noted in Section 2.2, the frequency-weighted expansion approach treats all admissions within an hour as occurring at the same angular point, thereby losing information about within-hour variability. Finer temporal resolution (e.g., quarter-hour or minute-level timestamps) would enable more granular characterization of arrival patterns.

Finally, our analysis is descriptive and does not establish causal mechanisms underlying the observed circadian rhythm. Factors such as work schedules, clinic hours, traffic patterns, and social routines likely drive the afternoon peak, but our data do not permit testing of such hypotheses. Nonetheless, the documented pattern provides a robust empirical foundation for operational planning, even in the absence of causal explanation.

## 4. CONCLUSION

By applying circular statistical techniques to 142,005 hourly ED admissions from a large Iowa hospital spanning four years, we demonstrate that patient arrivals follow a statistically significant, non-uniform 24-hour pattern characterized by an early-to-mid-afternoon mean, a late-afternoon modal peak, and broad dispersion across the day. Circular measures of central tendency and dispersion, together with the von Mises, cardioid, and wrapped normal distributions, provide a mathematically appropriate and interpretable framework for describing these rhythms.

Both the Rayleigh and Kuiper tests confirm that arrivals are far from uniformly distributed over the 24-hour cycle. Fitted circular distributions substantially outperform the circular uniform model in terms of AIC, BIC, CAIC, and HQIC. Visualizations such as the rose diagram, circular summary plot, CDF comparisons, and circular heatmap effectively communicate these findings to non-specialist audiences. More generally, this study reinforces a fundamental principle discussed throughout the literature on circular statistics [10, 11, 14, 15, 17]: variables that in their essence involve circularity should be treated as such.

More broadly, this study underscores a general principle highlighted in the circular statistics literature [10,11,14,15,17]: when variables are inherently cyclical, analyses should respect their circular structure. For temporal patterns in emergency care, circular methods offer a practical complement to established forecasting approaches and can strengthen evidence-based decisions about staffing, capacity, and resource deployment in EDs.

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