

SUPPLEMENTARY TABLES

Table S1: True Values of the Parameters used in Scenario one, Where $\beta_{.1}$, $\beta_{.2}$, $\beta_{.3}$ Correspond to Variables Age, Sex, and trt, respectively

	GH-1	GH-2	GH-3	GH-4	PH-1	PH-2	AFT-1	AFT-2	AH-1	AH-2
η	1.25	1.25	1.25	1.25	1.25	1.25	1.45	1.45	1.25	1.25
ν	2.45	2.45	2.45	2.45	2.20	2.20	2.10	2.10	2.20	2.20
δ	1.15	1.15	1.15	1.15	1.15	1.15	1.15	1.15	1.15	1.15
β_{11}	0.25	-0.25	-0.25	0.25	-	-	0.10	-0.10	-0.25	0.25
β_{12}	0.35	-0.35	-0.35	0.35	-	-	0.15	-0.15	0.35	0.35
β_{13}	0.15	-0.15	-0.15	0.15	-	-	0.20	-0.20	-0.15	0.15
β_{21}	0.15	-0.15	0.15	-0.15	0.15	-0.15	0.10	-0.10	-	-
β_{22}	0.10	-0.10	0.10	-0.10	0.10	-0.10	0.15	-0.15	-	-
β_{23}	0.25	-0.25	0.25	-0.25	0.25	-0.25	0.20	-0.20	-	-
r (20%)	0.18	0.18	0.18	0.18	0.180	0.180	0.175	0.175	0.15	0.15
r (40%)	0.48	0.48	0.48	0.48	0.45	0.45	0.40	0.40	0.39	0.39

COMPARISON OF MODELS BY INFORMATION CRITERIA

Table S2: Information Criteria for Scenario one from the GH-1 Structure

Model	n=1000				n=5000			
	AIC	AICc	BIC	HQIC	AIC	AICc	BIC	HQIC
20% Censoring rate					20% Censoring rate			
GH-BXII	1708.017*	1708.198	1752.186	1724.804	8228.555	8228.591	8287.210	8249.113
PH-BXII	1747.959	1748.044	1777.406	1759.151	8396.410	8396.427	8435.513	8410.115
AFT-BXII	1718.413	1718.479	1747.859	1729.605	8289.004	8289.021	8328.108	8302.710
AH-BXII	1785.405	1785.490	1814.852	1796.597	8601.212	8601.229	8640.315	8614.917
40% Censoring rate					40% Censoring rate			
GH-BXII	1296.560	1296.741	1340.730	1313.347	6280.410	6280.446	6339.065	6300.967
PH-BXII	1316.477	1316.561	1345.923	1327.669	6393.880	6393.897	6432.983	6407.585
AFT-BXII	1388.193	1388.277	1417.639	1399.385	6729.293	6729.310	6768.397	6742.998
AH-BXII	1414.168	1414.253	1443.615	1425.360	6853.126	6853.142	6892.229	6866.831

*The lowest values in the table are bolded.

Table S3: Information Criteria for Scenario Two from the PH-1 Structure

Model	n=1000				n=5000			
	AIC	AICc	BIC	HQIC	AIC	AICc	BIC	HQIC
20% Censoring rate					20% Censoring rate			
GH-BXII	1858.224	1858.406	1902.394	1875.012	9009.343	9009.379	9067.998	9029.901
PH-BXII	1855.148	1855.232	1884.594	1866.34	9006.204	9006.220	9054.307	9019.909
AFT-BXII	1860.01	1860.094	1889.456	1871.201	9029.432	9029.449	9068.536	9043.137
AH-BXII	1878.152	1878.236	1907.598	1889.343	9119.406	9119.423	9158.509	9133.111
40% Censoring rate					40% Censoring rate			
GH-BXII	1440.848	1441.029	1485.018	1457.635	7018.001	7018.037	7076.656	7038.559
PH-BXII	1437.813	1437.897	1467.259	1449.005	7014.885	7014.901	7053.988	7028.590
AFT-BXII	1518.846	1518.93	1548.292	1530.037	7404.185	7404.201	7443.288	7417.89
AH-BXII	1530.792	1530.876	1560.238	1541.983	7463.936	7463.952	7503.039	7477.641

Table S4: Information Criteria for Scenario Three from the AFT-1 Structure

Model	n=1000				n=5000			
	AIC	AICc	BIC	HQIC	AIC	AICc	BIC	HQIC
20% Censoring rate					20% Censoring rate			
GH-BXII	2046.954	2047.136	2091.124	2063.742	9999.467	9999.503	10058.120	10020.020
AFT-BXII	2044.057	2044.141	2073.504	2055.249	9996.403	9996.420	10035.510	10010.110
PH-BXII	2050.242	2050.327	2079.689	2061.434	10026.470	10026.490	10065.580	10040.180
AH-BXII	2084.260	2084.345	2113.707	2095.452	10077.280	10077.300	10116.380	10090.99
40% Censoring rate					40% Censoring rate			
GH-BXII	1614.13	1614.312	1658.300	1630.918	7906.328	7906.364	7964.982	7926.885
AFT-BXII	1611.251	1611.335	1640.697	1622.443	7903.307	7903.324	7942.411	7917.012
PH-BXII	1615.464	1615.548	1644.910	1626.656	7924.696	7924.713	7963.799	7938.401
AH-BXII	1621.403	1621.488	1650.850	1632.595	7954.646	7954.663	7993.749	7968.351

Table S5: Information Criteria for Scenario Four from the AH-1 Structure

Model	n=1000				n=5000			
	AIC	AICc	BIC	HQIC	AIC	AICc	BIC	HQIC
20% Censoring rate					20% Censoring rate			
GH-BXII	2110.185	2110.367	2154.355	2126.973	10310.730	10310.760	10369.380	10331.290
AH-BXII	2107.323	2107.407	2136.769	2118.515	10307.700	10307.720	10346.810	10321.410
PH-BXII	2134.187	2134.272	2163.634	2145.379	10448.170	10448.190	10487.270	10461.880
AFT-BXII	2121.826	2121.911	2151.273	2133.018	10384.470	10384.490	10423.570	10398.180
40% Censoring rate					40% Censoring rate			
GH-BXII	1630.528	1630.710	1674.698	1647.316	7989.372	7989.408	8048.027	8009.930
AH-BXII	1627.670	1627.755	1657.117	1638.862	7986.378	7986.395	8025.481	8000.083
PH-BXII	1646.223	1646.307	1675.669	1657.414	8083.656	8083.672	8122.759	8097.361
AFT-BXII	1636.784	1636.869	1666.231	1647.976	8033.936	8033.953	8073.040	8047.641

2. PERFORMANCE OF THE MLES

2.1 GH-2 structure

$$h_{BXII}^{AFT}(t; x_j) = h_{BXII}(t \times \exp(-0.10 \times \text{age} - 0.15 \times \text{sex} - 0.20 \times \text{TRT}); 1.45, 2.10, 1.15) \\ \times \exp(-0.10 \times \text{age} - 0.15 \times \text{sex} - 0.20 \times \text{TRT}).$$

Table S6: Simulation from GH-2 Structure with $(\eta, \nu, \delta) = (1.25, 2.45, 1.15)$, $B_t = (-0.25, -0.35, -0.15)$, $B_h = (-0.15, -0.10, -0.25)$ and $N = 1000$ ($N=1000$ Datasets).

Model	Parameters	TRUE	M.MLE	m.MLE	E.SD	M.SE	coverage	Bias	MSE	RMSE
20% Censoring										
GH-BXII	η	1.250	1.208	1.201	0.114	0.122	0.944	-0.042	0.015	0.122
	ν	2.450	2.455	2.453	0.052	0.053	0.952	0.005	0.003	0.052
	δ	1.150	1.061	1.042	0.164	0.193	0.969	-0.089	0.035	0.187
	β_{11}	-0.250	-0.249	-0.250	0.058	0.058	0.947	0.001	0.003	0.058
	β_{12}	-0.350	-0.345	-0.337	0.124	0.117	0.945	0.005	0.015	0.124
	β_{13}	-0.150	-0.158	-0.159	0.117	0.115	0.940	-0.008	0.014	0.118
	β_{21}	-0.150	-0.153	-0.153	0.047	0.047	0.950	-0.003	0.002	0.047

	β_{22}	-0.100	-0.107	-0.104	0.094	0.093	0.949	-0.007	0.009	0.094
	β_{23}	-0.250	-0.244	-0.247	0.096	0.091	0.950	0.006	0.009	0.096
40% Censoring										
GH-BXII	η	1.250	1.277	1.252	0.187	0.166	0.939	0.027	0.036	0.189
	ν	2.450	2.416	2.413	0.076	0.060	0.849	-0.034	0.007	0.084
	δ	1.150	1.177	1.117	0.309	0.273	0.948	0.027	0.096	0.310
	β_{11}	-0.250	-0.256	-0.254	0.078	0.080	0.957	-0.006	0.006	0.079
	β_{12}	-0.350	-0.357	-0.343	0.249	0.434	0.953	-0.007	0.062	0.249
	β_{13}	-0.150	-0.166	-0.159	0.166	0.157	0.945	-0.016	0.028	0.167
	β_{21}	-0.150	-0.146	-0.146	0.072	0.070	0.940	0.004	0.005	0.072
	β_{22}	-0.100	-0.094	-0.109	0.233	0.437	0.971	0.006	0.054	0.233
	β_{23}	-0.250	-0.237	-0.234	0.143	0.134	0.944	0.013	0.021	0.144

MMLE: mean of the MLEs; Mmle: median of the MLEs; E.SD: empirical standard deviation; M.SE: model-based standard error; MSE: mean-square error; RMSE: root-mean-square error; Coverage: coverage proportions.

Table S7: Simulation from GH-2 Structure with $(\eta, \nu, \delta) = (1.25, 2.45, 1.15)$, $B_t = (-0.25, -0.35, -0.15)$, $B_h = (-0.15, -0.10, -0.25)$ and $N = 1000$ and $N=1000$ Samples ($N=1000$ Datasets)

Model	Parameters	TRUE	M.MLE	m.MLE	E.SD	M.SE	Coverage	Bias	MSE	RMSE
20% Censoring										
GH-BXII	η	1.250	1.229	1.227	0.050	0.055	0.958	-0.021	0.003	0.054
	ν	2.450	2.471	2.470	0.022	0.024	0.879	0.021	0.001	0.030
	δ	1.150	1.113	1.106	0.073	0.089	0.968	-0.037	0.007	0.082
	β_{11}	-0.250	-0.246	-0.246	0.026	0.026	0.949	0.004	0.001	0.026
	β_{12}	-0.350	-0.347	-0.347	0.049	0.052	0.964	0.003	0.002	0.049
	β_{13}	-0.150	-0.151	-0.151	0.049	0.051	0.955	-0.001	0.002	0.049
	β_{21}	-0.150	-0.153	-0.154	0.021	0.021	0.954	-0.003	0.000	0.022
	β_{22}	-0.100	-0.102	-0.103	0.041	0.042	0.953	-0.002	0.002	0.041
	β_{23}	-0.250	-0.249	-0.248	0.041	0.041	0.958	0.001	0.002	0.041
40% Censoring										
GH-BXII	η	1.250	1.248	1.246	0.072	0.072	0.954	-0.002	0.005	0.072
	ν	2.450	2.458	2.457	0.033	0.027	0.888	0.008	0.001	0.034
	δ	1.150	1.144	1.138	0.112	0.119	0.969	-0.006	0.012	0.112
	β_{11}	-0.250	-0.249	-0.249	0.035	0.034	0.955	0.001	0.001	0.035
	β_{12}	-0.350	-0.349	-0.349	0.067	0.069	0.957	0.001	0.005	0.067
	β_{13}	-0.150	-0.151	-0.150	0.065	0.067	0.959	-0.001	0.004	0.065
	β_{21}	-0.150	-0.151	-0.152	0.030	0.030	0.954	-0.001	0.001	0.030
	β_{22}	-0.100	-0.100	-0.102	0.060	0.060	0.958	0.000	0.004	0.060
	β_{23}	-0.250	-0.250	-0.251	0.056	0.058	0.961	0.000	0.003	0.056

MMLE: mean of the MLEs; Mmle: median of the MLEs; E.SD: empirical standard deviation; M.SE: model-based standard error; MSE: mean-square error; RMSE: root-mean-square error; Coverage: coverage proportions.2.2 GH-3 structure.

$$h_{BXII}^{AFT}(t; x_j) = h_{BXII}(t \times \exp(0.10 \times age + 0.15 \times sex + 0.20 \times TRT); 1.45, 2.10, 1.15) \times \exp(-0.10 \times age - 0.15 \times sex - 0.20 \times TRT).$$

Table S8: Simulation from GH-3 Structure with $(\eta, \nu, \delta) = (1.25, 2.45, 1.15)$, $B_t = (-0.25, -0.35, -0.15)$, $B_h = (0.15, 0.10, 0.25)$ and $N = 1000$ ($N=1000$ Datasets)

Model	Parameters	TRUE	M.MLE	m.MLE	E.SD	M.SE	coverage	Bias	MSE	RMSE
20% Censoring										
GH-BXII	η	1.250	1.231	1.222	0.120	0.122	0.953	-0.019	0.015	0.121

	ν	2.450	2.434	2.436	0.052	0.050	0.919	-0.016	0.003	0.055
	δ	1.150	1.096	1.074	0.181	0.198	0.961	-0.054	0.036	0.189
	β_{11}	-0.250	-0.252	-0.249	0.058	0.061	0.963	-0.002	0.003	0.058
	β_{12}	-0.350	-0.351	-0.346	0.130	0.122	0.942	-0.001	0.017	0.130
	β_{13}	-0.150	-0.155	-0.152	0.116	0.120	0.964	-0.005	0.014	0.116
	β_{21}	0.150	0.146	0.145	0.052	0.055	0.953	-0.004	0.003	0.052
	β_{22}	0.100	0.095	0.092	0.108	0.106	0.945	-0.005	0.012	0.108
	β_{23}	0.250	0.246	0.237	0.104	0.104	0.959	-0.004	0.011	0.104
40% Censoring										
GH-BXII	η	1.250	1.279	1.251	0.235	0.163	0.928	0.029	0.056	0.236
	ν	2.450	2.420	2.420	0.075	0.056	0.819	-0.030	0.006	0.080
	δ	1.150	1.189	1.116	0.510	0.275	0.943	0.039	0.261	0.511
	β_{11}	-0.250	-0.255	-0.252	0.078	0.082	0.972	-0.005	0.006	0.078
	β_{12}	-0.350	-0.351	-0.343	0.173	0.165	0.960	-0.001	0.030	0.173
	β_{13}	-0.150	-0.159	-0.152	0.166	0.160	0.952	-0.009	0.028	0.166
	β_{21}	0.150	0.151	0.149	0.075	0.078	0.961	0.001	0.006	0.075
	β_{22}	0.100	0.097	0.089	0.155	0.154	0.960	-0.003	0.024	0.155
	β_{23}	0.250	0.254	0.247	0.156	0.148	0.962	0.004	0.024	0.156

MMLE: mean of the MLEs; Mmle: median of the MLEs; E.SD: empirical standard deviation; M.SE: model-based standard error; MSE: mean-square error; RMSE: root-mean-square error; Coverage: coverage proportions.

Table S9: Simulation from GH-3 Structure with $(\eta, \nu, \delta) = (1.25, 2.45, 1.15)$, $B_t = (-0.25, -0.35, -0.15)$, $B_h = (0.15, 0.10, 0.25)$ and $N = 5000$ ($N=1000$ Datasets)

Model	Parameters	TRUE	M.MLE	m.MLE	E.SD	M.SE	Coverage	Bias	MSE	RMSE
20% Censoring										
GH-BXII	η	1.250	1.238	1.236	0.053	0.055	0.960	-0.012	0.003	0.054
	ν	2.450	2.462	2.461	0.023	0.022	0.908	0.012	0.001	0.026
	δ	1.150	1.125	1.120	0.080	0.090	0.961	-0.025	0.007	0.084
	β_{11}	-0.250	-0.247	-0.247	0.026	0.027	0.959	0.003	0.001	0.026
	β_{12}	-0.350	-0.346	-0.346	0.051	0.054	0.971	0.004	0.003	0.051
	β_{13}	-0.150	-0.149	-0.148	0.051	0.053	0.956	0.001	0.003	0.051
	β_{21}	0.150	0.147	0.147	0.023	0.025	0.957	-0.003	0.001	0.024
	β_{22}	0.100	0.097	0.097	0.046	0.048	0.959	-0.003	0.002	0.046
	β_{23}	0.250	0.249	0.250	0.047	0.047	0.950	-0.001	0.002	0.047
40% Censoring										
GH-BXII	η	1.250	1.246	1.242	0.071	0.070	0.944	-0.004	0.005	0.071
	ν	2.450	2.458	2.458	0.033	0.025	0.855	0.008	0.001	0.034
	δ	1.150	1.139	1.133	0.113	0.119	0.958	-0.011	0.013	0.113
	β_{11}	-0.250	-0.247	-0.247	0.034	0.035	0.954	0.003	0.001	0.034
	β_{12}	-0.350	-0.350	-0.348	0.067	0.070	0.957	0.000	0.004	0.067
	β_{13}	-0.150	-0.149	-0.148	0.065	0.068	0.962	0.001	0.004	0.065
	β_{21}	0.150	0.148	0.146	0.033	0.034	0.961	-0.002	0.001	0.033
	β_{22}	0.100	0.102	0.102	0.065	0.066	0.953	0.002	0.004	0.065
	β_{23}	0.250	0.249	0.248	0.062	0.064	0.957	-0.001	0.004	0.062

MMLE: mean of the MLEs; Mmle: median of the MLEs; E.SD: empirical standard deviation; M.SE: model-based standard error; MSE: mean-square error; RMSE: root-mean-square error; Coverage: coverage proportions.

2.3 GH-4 structure

$$h_{BXII}^{AFT}(t; x_j) = h_{BXII}(t \times \exp(-0.10 \times \text{age} - 0.15 \times \text{sex} - 0.20 \times \text{TRT}); 1.45, 2.10, 1.15) \\ \times \exp(0.10 \times \text{age} + 0.15 \times \text{sex} + 0.20 \times \text{TRT}).$$

Table S10: Simulation from GH-4 Structure with $(\eta, \nu, \delta) = (1.25, 2.45, 1.15)$, $B_t = (0.25, 0.35, 0.15)$, $B_h = (-0.15, -0.10, -0.25)$ and $N = 1000$ (N=1000 Datasets)

Model	Parameters	TRUE	M.MLE	m.MLE	E.SD	M.SE	Coverage	Bias	MSE	RMSE
20% Censoring										
GH-BXII	η	1.250	1.224	1.215	0.113	0.122	0.959	-0.026	0.013	0.116
	ν	2.450	2.438	2.435	0.059	0.056	0.931	-0.012	0.004	0.061
	δ	1.150	1.082	1.067	0.156	0.190	0.977	-0.068	0.029	0.170
	β_{11}	0.250	0.241	0.240	0.051	0.053	0.969	-0.009	0.003	0.051
	β_{12}	0.350	0.345	0.347	0.107	0.105	0.945	-0.005	0.011	0.107
	β_{13}	0.150	0.138	0.137	0.107	0.104	0.942	-0.012	0.012	0.107
	β_{21}	-0.150	-0.142	-0.141	0.039	0.041	0.958	0.008	0.002	0.039
	β_{22}	-0.100	-0.092	-0.089	0.077	0.079	0.962	0.008	0.006	0.077
	β_{23}	-0.250	-0.241	-0.241	0.080	0.078	0.947	0.009	0.006	0.080
40% Censoring										
GH-BXII	η	1.250	1.288	1.275	0.177	0.161	0.936	0.038	0.033	0.181
	ν	2.450	2.402	2.401	0.079	0.062	0.820	-0.048	0.009	0.093
	δ	1.150	1.185	1.147	0.275	0.257	0.951	0.035	0.077	0.277
	β_{11}	0.250	0.248	0.248	0.065	0.068	0.961	-0.002	0.004	0.065
	β_{12}	0.350	0.355	0.360	0.136	0.133	0.945	0.005	0.018	0.136
	β_{13}	0.150	0.140	0.145	0.137	0.131	0.939	-0.010	0.019	0.137
	β_{21}	-0.150	-0.146	-0.144	0.054	0.054	0.949	0.004	0.003	0.054
	β_{22}	-0.100	-0.101	-0.099	0.103	0.102	0.953	-0.001	0.011	0.103
	β_{23}	-0.250	-0.244	-0.244	0.104	0.101	0.947	0.006	0.011	0.104

MMLE: mean of the MLEs; Mmle: median of the MLEs; E.SD: empirical standard deviation; M.SE: model-based standard error; MSE: mean-square error; RMSE: root-mean-square error; Coverage: coverage proportions.

Table S11: Simulation from GH-4 Structure with $(\eta, \nu, \delta) = (1.25, 2.45, 1.15)$, $B_t = (0.25, 0.35, 0.15)$, $B_h = (-0.15, -0.10, -0.25)$ and $N = 5000$ (N=1000 Datasets)

Model	Parameters	TRUE	M.MLE	m.MLE	E.SD	M.SE	Coverage	Bias	MSE	RMSE
20% Censoring										
GH-BXII	η	1.250	1.234	1.233	0.048	0.055	0.969	-0.016	0.003	0.050
	ν	2.450	2.464	2.464	0.026	0.025	0.902	0.014	0.001	0.029
	δ	1.150	1.119	1.116	0.067	0.086	0.983	-0.031	0.005	0.074
	β_{11}	0.250	0.246	0.246	0.022	0.023	0.963	-0.004	0.000	0.022
	β_{12}	0.350	0.343	0.342	0.045	0.046	0.948	-0.007	0.002	0.045
	β_{13}	0.150	0.146	0.144	0.045	0.046	0.956	-0.004	0.002	0.046
	β_{21}	-0.150	-0.148	-0.148	0.018	0.018	0.955	0.002	0.000	0.018
	β_{22}	-0.100	-0.096	-0.097	0.035	0.035	0.941	0.004	0.001	0.035
	β_{23}	-0.250	-0.247	-0.246	0.034	0.035	0.958	0.003	0.001	0.034
40% Censoring										
GH-BXII	η	1.250	1.258	1.254	0.069	0.070	0.953	0.008	0.005	0.070
	ν	2.450	2.447	2.447	0.033	0.028	0.902	-0.003	0.001	0.033
	δ	1.150	1.157	1.149	0.106	0.113	0.966	0.007	0.011	0.106
	β_{11}	0.250	0.248	0.247	0.028	0.029	0.957	-0.002	0.001	0.028

	β_{12}	0.350	0.346	0.344	0.056	0.057	0.955	-0.004	0.003	0.056
	β_{13}	0.150	0.147	0.148	0.057	0.057	0.950	-0.003	0.003	0.057
	β_{21}	-0.150	-0.150	-0.150	0.023	0.024	0.955	0.000	0.001	0.023
	β_{22}	-0.100	-0.099	-0.101	0.045	0.045	0.948	0.001	0.002	0.045
	β_{23}	-0.250	-0.249	-0.248	0.044	0.045	0.959	0.001	0.002	0.044

MMLE: mean of the MLEs; Mmle: median of the MLEs; E.SD: empirical standard deviation; M.SE: model-based standard error; MSE: mean-square error; RMSE: root-mean-square error; Coverage: coverage proportions.

2.4. PH-2 structure

$$h_{BXII}^{PH}(t; x_j) = h_{BXII}(t; 1.25, 2.20, 1.15) \times \exp(-0.15 \times age - 0.10 \times sex - 0.25 \times TRT).$$

Table S12: Information Criteria for Scenario 2 from the PH-2 Structure (N=1000 Datasets)

n=1000					n=5000			
Model	AIC	AICc	BIC	HQIC	AIC	AICc	BIC	HQIC
20% Censoring rate					20% Censoring rate			
GH-BXII	2154.734	2154.916	2198.904	2171.522	10580.876	10580.912	10639.531	10601.434
PH-BXII	2151.911	2151.996	2181.358	2163.103	10577.694	10577.711	10616.797	10591.399
AFT-BXII	2156.777	2156.862	2186.224	2167.969	10605.200	10605.210	10644.300	10618.900
AH-BXII	2175.967	2176.052	2205.414	2187.159	10704.770	10704.780	10743.87	10718.47
40% Censoring rate					40% Censoring rate			
GH-BXII	1583.561	1583.743	1627.731	1600.349	7785.661	7785.697	7844.316	7806.219
PH-BXII	1580.716	1580.800	1610.162	1591.907	7782.612	7782.629	7821.715	7796.317
AFT-BXII	1687.060	1687.144	1716.506	1698.252	8307.889	8307.906	8346.992	8321.594
AH-BXII	1699.643	1699.727	1729.089	1710.835	8374.575	8374.592	8413.678	8388.280

Table S13: Simulation from PH-2 Structure with $(\eta, \nu, \delta) = (1.25, 2.20, 1.15)$, $B = (-0.15, -0.10, -0.25)$ and $N = 1000$

Model	Parameters	TRUE	M.MLE	m.MLE	E.SD	M.SE	Coverage	Bias	MSE	RMSE
20% Censoring										
GH-BXII	η	1.250	1.211	1.204	0.129	0.138	0.964	-0.039	0.018	0.134
	ν	2.200	2.200	2.200	0.046	0.055	0.977	0.000	0.002	0.046
	δ	1.150	1.070	1.053	0.163	0.196	0.974	-0.080	0.033	0.181
	β_{11}	0	-0.008	-0.007	0.062	0.062	0.948	-0.008	0.004	0.062
	β_{12}	0	-0.002	0.000	0.130	0.123	0.945	-0.002	0.017	0.130
	β_{13}	0	-0.011	-0.009	0.128	0.124	0.946	-0.011	0.016	0.128
	β_{21}	-0.150	-0.145	-0.145	0.040	0.040	0.952	0.005	0.002	0.040
	β_{22}	-0.100	-0.097	-0.096	0.078	0.079	0.953	0.003	0.006	0.078
	β_{23}	-0.250	-0.244	-0.247	0.082	0.079	0.952	0.006	0.007	0.082
PH-BXII	η	1.250	1.218	1.212	0.070	0.104	0.995	-0.032	0.006	0.077
	ν	2.200	2.192	2.190	0.047	0.054	0.967	-0.008	0.002	0.048
	δ	1.150	1.073	1.061	0.106	0.158	0.990	-0.077	0.017	0.131
	β_1	-0.150	-0.147	-0.145	0.037	0.038	0.958	0.003	0.001	0.037
	β_2	-0.100	-0.097	-0.097	0.073	0.075	0.952	0.003	0.005	0.073
	β_3	-0.250	-0.246	-0.247	0.075	0.075	0.952	0.004	0.006	0.075
40% Censoring										
GH-BXII	η	1.250	1.280	1.261	0.199	0.181	0.945	0.030	0.040	0.201
	ν	2.200	2.167	2.164	0.064	0.060	0.892	-0.033	0.005	0.073
	δ	1.150	1.172	1.131	0.285	0.265	0.954	0.022	0.081	0.285

	β_{11}	0	-0.008	-0.005	0.080	0.081	0.954	-0.008	0.006	0.080
	β_{12}	0	0.000	0.002	0.167	0.160	0.947	0.000	0.028	0.167
	β_{13}	0	-0.017	-0.011	0.168	0.160	0.948	-0.017	0.028	0.168
	β_{21}	-0.150	-0.144	-0.143	0.055	0.053	0.937	0.006	0.003	0.055
	β_{22}	-0.100	-0.100	-0.096	0.107	0.105	0.955	0.000	0.011	0.107
	β_{23}	-0.250	-0.241	-0.238	0.109	0.105	0.947	0.009	0.012	0.109
PH-BXII	η	1.250	1.283	1.269	0.123	0.137	0.977	0.033	0.016	0.128
	ν	2.200	2.157	2.159	0.064	0.060	0.871	-0.043	0.006	0.076
	δ	1.150	1.162	1.144	0.184	0.207	0.980	0.012	0.034	0.184
	β_1	-0.150	-0.147	-0.144	0.044	0.044	0.944	0.003	0.002	0.044
	β_2	-0.100	-0.099	-0.097	0.089	0.088	0.945	0.001	0.008	0.089
	β_3	-0.250	-0.247	-0.247	0.089	0.088	0.946	0.003	0.008	0.089

MMLE: mean of the MLEs; Mmle: median of the MLEs; E.SD: empirical standard deviation; M.SE: model-based standard error; MSE: mean-square error; RMSE: root-mean-square error; Coverage: coverage proportions.

Table S14: Simulation from PH-2 Structure with $(\eta, \nu, \delta) = (1.25, 2.20, 1.15)$, $B = (-0.15, -0.10, -0.25)$ and $N = 5000$

Model	Parameters	TRUE	M.MLE	m.MLE	E.SD	M.SE	Coverage	Bias	MSE	RMSE
20% Censoring										
GH-BXII	η	1.250	1.225	1.222	0.054	0.062	0.972	-0.025	0.004	0.060
	ν	2.200	2.220	2.220	0.019	0.024	0.927	0.020	0.001	0.028
	δ	1.150	1.110	1.106	0.070	0.089	0.978	-0.040	0.007	0.081
	β_{11}	0	-0.001	0.000	0.026	0.027	0.958	-0.001	0.001	0.026
	β_{12}	0	-0.002	-0.004	0.053	0.055	0.962	-0.002	0.003	0.053
	β_{13}	0	-0.002	-0.001	0.053	0.055	0.960	-0.002	0.003	0.053
	β_{21}	-0.150	-0.150	-0.150	0.018	0.018	0.943	0.000	0.000	0.018
	β_{22}	-0.100	-0.099	-0.100	0.036	0.035	0.948	0.001	0.001	0.036
	β_{23}	-0.250	-0.249	-0.248	0.035	0.036	0.958	0.001	0.001	0.035
PH-BXII	η	1.250	1.228	1.227	0.031	0.047	0.988	-0.022	0.001	0.038
	ν	2.200	2.219	2.218	0.021	0.024	0.924	0.019	0.001	0.028
	δ	1.150	1.112	1.112	0.048	0.072	0.986	-0.038	0.004	0.061
	β_1	-0.150	-0.150	-0.150	0.017	0.017	0.950	0.000	0.000	0.017
	β_2	-0.100	-0.099	-0.100	0.034	0.033	0.936	0.001	0.001	0.034
	β_3	-0.250	-0.249	-0.250	0.033	0.033	0.957	0.001	0.001	0.033
40% Censoring										
GH-BXII	η	1.250	1.249	1.244	0.077	0.078	0.953	-0.001	0.006	0.077
	ν	2.200	2.206	2.205	0.027	0.027	0.947	0.006	0.001	0.027
	δ	1.150	1.145	1.137	0.107	0.116	0.973	-0.005	0.011	0.107
	β_{11}	0	-0.002	-0.002	0.035	0.035	0.959	-0.002	0.001	0.035
	β_{12}	0	-0.003	-0.001	0.068	0.069	0.956	-0.003	0.005	0.068
	β_{13}	0	-0.003	-0.003	0.068	0.069	0.956	-0.003	0.005	0.068
	β_{21}	-0.150	-0.149	-0.149	0.023	0.023	0.942	0.001	0.001	0.023
	β_{22}	-0.100	-0.098	-0.100	0.047	0.046	0.944	0.002	0.002	0.047
	β_{23}	-0.250	-0.249	-0.249	0.046	0.046	0.958	0.001	0.002	0.046
PH-BXII	η	1.250	1.252	1.252	0.048	0.060	0.984	0.002	0.002	0.048
	ν	2.200	2.204	2.202	0.027	0.027	0.948	0.004	0.001	0.027
	δ	1.150	1.147	1.143	0.072	0.092	0.985	-0.003	0.005	0.072
	β_1	-0.150	-0.150	-0.151	0.019	0.020	0.950	0.000	0.000	0.019

	β_2	-0.100	-0.099	-0.100	0.040	0.039	0.937	0.001	0.002	0.040
	β_3	-0.250	-0.250	-0.251	0.038	0.039	0.956	0.000	0.001	0.038

MMLE: mean of the MLEs; Mmle: median of the MLEs; E.SD: empirical standard deviation; M.SE: model-based standard error; MSE: mean-square error; RMSE: root-mean-square error; Coverage: coverage proportions.

2.5. AFT-2 structure

$$h_{BXII}^{AFT}(t; x_j) = h_{BXII}(t \times \exp(-0.10 \times age - 0.15 \times sex - 0.20 \times TRT); 1.45, 2.10, 1.15) \times \exp(-0.10 \times age - 0.15 \times sex - 0.20 \times TRT).$$

Table S15: Information Criteria for Scenario 2 from the AFT-2 Structure (N=1000 Datasets)

	N=1000				N=5000			
Model	AIC	AICc	BIC	HQIC	AIC	AICc	BIC	HQIC
20% Censoring rate					20% Censoring rate			
GH-BXII	2312.003	2312.185	2356.173	2328.791	11414.597	11414.633	11473.252	11435.155
AFT-BXII	2309.173	2309.257	2338.619	2320.364	11411.527	11411.543	11450.630	11425.232
PH-BXII	2314.346	2314.431	2343.793	2325.538	11435.750	11435.770	11474.850	11449.461
AH-BXII	2321.327	2321.411	2350.773	2332.518	11474.640	11474.660	11513.740	11488.340
40% Censoring rate					40% Censoring rate			
GH-BXII	1717.143	1717.325	1761.313	1733.931	8482.107	8482.143	8540.762	8502.665
AFT-BXII	1714.363	1714.447	1743.809	1725.555	8479.048	8479.064	8518.151	8492.753
PH-BXII	1717.688	1717.772	1747.134	1728.879	8494.972	8494.989	8534.076	8508.677
AH-BXII	1720.427	1720.511	1749.873	1731.619	8513.014	8513.031	8552.117	8526.719

Table S16: Simulation from AFT-2 Structure with $(\eta, \nu, \delta) = (1.45, 2.10, 1.15)$, $B = (-0.10, -0.15, -0.20)$ and $N = 1000$.

Model	Parameters	TRUE	M.MLE	m.MLE	E.SD	M.SE	Coverage	Bias	MSE	RMSE
20% Censoring										
GH-BXII	η	1.450	1.402	1.390	0.165	0.153	0.910	-0.048	0.029	0.172
	ν	2.100	2.103	2.102	0.045	0.055	0.984	0.003	0.002	0.045
	δ	1.150	1.071	1.048	0.177	0.211	0.970	-0.079	0.038	0.194
	β_{11}	-0.100	-0.104	-0.105	0.071	0.072	0.949	-0.004	0.005	0.071
	β_{12}	-0.150	-0.151	-0.147	0.148	0.143	0.942	-0.001	0.022	0.148
	β_{13}	-0.200	-0.206	-0.207	0.149	0.143	0.941	-0.006	0.022	0.149
	β_{21}	-0.100	-0.099	-0.097	0.043	0.043	0.946	0.001	0.002	0.043
	β_{22}	-0.150	-0.149	-0.147	0.084	0.086	0.961	0.001	0.007	0.084
	β_{23}	-0.200	-0.198	-0.201	0.090	0.086	0.945	0.002	0.008	0.090
AFT-BXII	η	1.450	1.396	1.390	0.109	0.129	0.970	-0.054	0.015	0.122
	ν	2.100	2.100	2.099	0.044	0.055	0.987	0.000	0.002	0.044
	δ	1.150	1.057	1.047	0.102	0.171	0.995	-0.093	0.019	0.139
	β_1	-0.100	-0.100	-0.100	0.027	0.028	0.959	0.000	0.001	0.027
	β_2	-0.150	-0.150	-0.149	0.056	0.056	0.955	0.000	0.003	0.056
	β_3	-0.200	-0.201	-0.201	0.055	0.056	0.952	-0.001	0.003	0.055
40% Censoring										
GH-BXII	η	1.450	1.490	1.462	0.251	0.199	0.890	0.040	0.065	0.254
	ν	2.100	2.070	2.070	0.062	0.061	0.915	-0.030	0.005	0.069
	δ	1.150	1.180	1.131	0.304	0.283	0.955	0.030	0.093	0.305
	β_{11}	-0.100	-0.107	-0.107	0.095	0.094	0.951	-0.007	0.009	0.095
	β_{12}	-0.150	-0.155	-0.153	0.200	0.190	0.946	-0.005	0.040	0.200

	β_{13}	-0.200	-0.222	-0.216	0.201	0.191	0.945	-0.022	0.041	0.202
	β_{21}	-0.100	-0.096	-0.096	0.061	0.059	0.947	0.004	0.004	0.061
	β_{22}	-0.150	-0.147	-0.146	0.119	0.118	0.962	0.003	0.014	0.119
	β_{23}	-0.200	-0.189	-0.190	0.126	0.119	0.953	0.011	0.016	0.126
AFT-BXII	η	1.450	1.481	1.465	0.179	0.167	0.938	0.031	0.033	0.182
	ν	2.100	2.068	2.065	0.059	0.060	0.922	-0.032	0.005	0.067
	δ	1.150	1.153	1.128	0.193	0.226	0.984	0.003	0.037	0.193
	β_1	-0.100	-0.100	-0.100	0.030	0.031	0.953	0.000	0.001	0.030
	β_2	-0.150	-0.150	-0.151	0.063	0.062	0.953	0.000	0.004	0.063
	β_3	-0.200	-0.202	-0.204	0.062	0.062	0.942	-0.002	0.004	0.062

MMLE: mean of the MLEs; Mmle: median of the MLEs; E.SD: empirical standard deviation; M.SE: model-based standard error; MSE: mean-square error; RMSE: root-mean-square error; Coverage: coverage proportions.

Table S17: Simulation from AFT-2 Structure with $(\eta, \nu, \delta) = (1.45, 2.10, 1.15)$, $B = (0.10, 0.15, 0.20)$ and $N = 5000$.

Model	Parameters	TRUE	M.MLE	m.MLE	E.SD	M.SE	Coverage	Bias	MSE	RMSE
20% Censoring										
GH-BXII	η	1.450	1.413	1.410	0.070	0.068	0.902	-0.037	0.006	0.079
	ν	2.100	2.124	2.123	0.018	0.025	0.911	0.024	0.001	0.030
	δ	1.150	1.103	1.099	0.076	0.095	0.967	-0.047	0.008	0.090
	β_{11}	-0.100	-0.098	-0.097	0.031	0.032	0.958	0.002	0.001	0.031
	β_{12}	-0.150	-0.149	-0.151	0.061	0.063	0.960	0.001	0.004	0.061
	β_{13}	-0.200	-0.197	-0.199	0.062	0.063	0.949	0.003	0.004	0.062
	β_{21}	-0.100	-0.101	-0.101	0.020	0.019	0.945	-0.001	0.000	0.020
	β_{22}	-0.150	-0.151	-0.151	0.038	0.038	0.951	-0.001	0.001	0.038
AFT-BXII	β_{23}	-0.200	-0.201	-0.201	0.038	0.039	0.950	-0.001	0.001	0.038
	η	1.450	1.409	1.408	0.047	0.058	0.947	-0.041	0.004	0.062
	ν	2.100	2.123	2.123	0.018	0.025	0.915	0.023	0.001	0.029
	δ	1.150	1.097	1.095	0.045	0.078	0.987	-0.053	0.005	0.070
	β_1	-0.100	-0.100	-0.100	0.012	0.012	0.944	0.000	0.000	0.012
	β_2	-0.150	-0.150	-0.150	0.025	0.024	0.942	0.000	0.001	0.025
GH-BXII	β_3	-0.200	-0.200	-0.200	0.024	0.024	0.956	0.000	0.001	0.024
	40% Censoring									
	η	1.450	1.445	1.440	0.098	0.085	0.913	-0.005	0.010	0.098
	ν	2.100	2.108	2.109	0.026	0.027	0.955	0.008	0.001	0.027
	δ	1.150	1.141	1.132	0.115	0.123	0.970	-0.009	0.013	0.115
	β_{11}	-0.100	-0.100	-0.101	0.040	0.040	0.962	0.000	0.002	0.040
	β_{12}	-0.150	-0.152	-0.150	0.079	0.080	0.953	-0.002	0.006	0.079
	β_{13}	-0.200	-0.200	-0.203	0.079	0.080	0.957	0.000	0.006	0.079
	β_{21}	-0.100	-0.100	-0.100	0.025	0.026	0.959	0.000	0.001	0.025
	β_{22}	-0.150	-0.148	-0.150	0.051	0.051	0.945	0.002	0.003	0.051
AFT-BXII	β_{23}	-0.200	-0.200	-0.200	0.051	0.051	0.948	0.000	0.003	0.051
	η	1.450	1.443	1.438	0.071	0.072	0.955	-0.007	0.005	0.071
	ν	2.100	2.108	2.109	0.026	0.027	0.957	0.008	0.001	0.027
	δ	1.150	1.137	1.133	0.076	0.100	0.986	-0.013	0.006	0.077
	β_1	-0.100	-0.100	-0.100	0.013	0.013	0.954	0.000	0.000	0.013
	β_2	-0.150	-0.150	-0.150	0.027	0.027	0.944	0.000	0.001	0.027
GH-BXII	β_3	-0.200	-0.200	-0.199	0.026	0.027	0.960	0.000	0.001	0.026

MMLE: mean of the MLEs; Mmle: median of the MLEs; E.SD: empirical standard deviation; M.SE: model-based standard error; MSE: mean-square error; RMSE: root-mean-square error; Coverage: coverage proportions.

2.6 AH-2 structure

$$h_{BXII}^{AH}(t; x_j) = h_{BXII}(t \times \exp(0.25 \times \text{age} + 0.35 \times \text{sex} + 0.15 \times \text{TRT}); 1.25, 2.20, 1.15).$$

Table S18: Information Criteria for Scenario Four from the AH-2 Structure (N=1000 Datasets)

Model	n=1000				n=5000			
	AIC	AICc	BIC	HQIC	AIC	AICc	BIC	HQIC
20% Censoring rate					20% Censoring rate			
GH-BXII	2496.418	2496.599	2540.587	2513.205	10262.757	10262.793	10321.411	10283.314
AH-BXII	2097.211	2097.295	2126.658	2108.403	10259.756	10259.772	10298.859	10273.461
PH-BXII	2125.373	2125.458	2154.819	2136.565	10407.090	10407.110	10446.200	10420.800
AFT-BXII	2114.132	2114.216	2125.578	2125.324	10347.670	10347.680	10386.770	10361.370
40% Censoring rate					40% Censoring rate			
GH-BXII	1628.802	1628.984	1672.972	1645.590	7978.451	7978.487	8037.105	7999.008
AH-BXII	1625.869	1625.953	1655.315	1637.060	7975.372	7975.388	8014.475	7989.077
PH-BXII	1645.545	1645.670	1675.032	1656.777	8079.992	8080.008	8119.095	8093.697
AFT -BXII	1636.440	1636.525	1665.887	1647.632	8030.969	8030.986	8070.072	8044.674

Table S19: Simulation from AH-2 Structure with $(\eta, \nu, \delta) = (1.25, 2.20, 1.15)$, $B = (0.25, 0.35, 0.15)$ and $N = 1000$.

Model	Parameters	TRUE	M.MLE	m.MLE	E.SD	M.SE	Coverage	Bias	MSE	RMSE
20% Censoring										
GH-BXII	η	1.250	2.319	2.280	0.307	0.174	0.759	0.069	0.099	0.315
	ν	2.200	2.165	2.167	0.049	0.059	0.944	-0.035	0.004	0.060
	δ	1.150	1.181	1.143	0.232	0.253	0.970	0.031	0.055	0.234
	β_{11}	0.250	0.246	0.247	0.070	0.071	0.951	-0.004	0.005	0.070
	β_{12}	0.350	0.353	0.351	0.143	0.141	0.946	0.003	0.021	0.143
	β_{13}	0.150	0.147	0.146	0.143	0.138	0.950	-0.003	0.020	0.143
	β_{21}	0.000	0.001	0.003	0.044	0.045	0.956	0.001	0.002	0.044
	β_{22}	0.000	-0.001	0.003	0.086	0.088	0.959	-0.001	0.007	0.086
AH-BXII	β_{23}	0.000	0.000	-0.002	0.089	0.086	0.941	0.000	0.008	0.089
	η	1.250	1.203	1.197	0.117	0.128	0.951	-0.047	0.016	0.126
	ν	2.200	2.205	2.204	0.042	0.054	0.986	0.005	0.002	0.042
	δ	1.150	1.059	1.051	0.120	0.163	0.972	-0.091	0.023	0.151
	β_1	0.250	0.244	0.244	0.056	0.056	0.957	-0.006	0.003	0.057
	β_2	0.350	0.346	0.348	0.116	0.112	0.935	-0.004	0.013	0.116
GH-BXII	β_3	0.150	0.146	0.143	0.113	0.110	0.953	-0.004	0.013	0.113
	40% Censoring									
	η	1.250	1.255	1.237	0.172	0.164	0.955	0.005	0.030	0.172
	ν	2.200	2.179	2.178	0.057	0.058	0.933	-0.021	0.004	0.061
	δ	1.150	1.134	1.100	0.235	0.237	0.970	-0.016	0.055	0.235
	β_{11}	0.250	0.243	0.245	0.069	0.069	0.951	-0.007	0.005	0.070
	β_{12}	0.350	0.351	0.355	0.141	0.138	0.946	0.001	0.020	0.141
	β_{13}	0.150	0.143	0.144	0.143	0.136	0.932	-0.007	0.020	0.143
	β_{21}	0.000	0.005	0.006	0.045	0.046	0.950	0.005	0.002	0.046
AH-BXII	β_{22}	0.000	0.002	0.003	0.091	0.090	0.958	0.002	0.008	0.091
	β_{23}	0.000	0.003	0.002	0.092	0.088	0.936	0.003	0.008	0.092
AH-BXII	η	1.250	1.250	1.239	0.157	0.152	0.949	0.000	0.025	0.157

	ν	2.200	2.181	2.180	0.058	0.058	0.937	-0.019	0.004	0.061
	δ	1.150	1.120	1.104	0.173	0.196	0.974	-0.030	0.031	0.175
	β_1	0.250	0.246	0.246	0.063	0.064	0.954	-0.004	0.004	0.063
	β_2	0.350	0.352	0.353	0.131	0.127	0.945	0.002	0.017	0.131
	β_3	0.150	0.145	0.147	0.130	0.126	0.940	-0.005	0.017	0.130

MMLE: mean of the MLEs; Mmle: median of the MLEs; E.SD: empirical standard deviation; M.SE: model-based standard error; MSE: mean-square error; RMSE: root-mean-square error; Coverage: coverage proportions.

Table S20: Simulation from AH-2 Structure with $(\eta, \nu, \delta) = (1.25, 2.20, 1.15)$, $B = (0.25, 0.35, 0.15)$ and $N = 5000$

Model	Parameters	TRUE	M.MLE	m.MLE	E.SD	M.SE	Coverage	Bias	MSE	RMSE
20% Censoring										
GH-BXII	η	1.250	1.225	1.221	0.051	0.060	0.969	-0.025	0.003	0.056
	ν	2.200	2.220	2.220	0.018	0.024	0.934	0.020	0.001	0.027
	δ	1.150	1.109	1.104	0.064	0.085	0.985	-0.041	0.006	0.076
	β_{11}	0.250	0.248	0.248	0.024	0.025	0.956	-0.002	0.001	0.025
	β_{12}	0.350	0.345	0.345	0.049	0.051	0.958	-0.005	0.002	0.049
	β_{13}	0.150	0.149	0.150	0.050	0.050	0.951	-0.001	0.003	0.050
	β_{21}	0.000	0.001	0.001	0.017	0.017	0.952	0.001	0.000	0.017
	β_{22}	0.000	0.002	0.002	0.033	0.033	0.946	0.002	0.001	0.033
	β_{23}	0.000	0.001	0.002	0.033	0.033	0.949	0.001	0.001	0.033
AH-BXII	η	1.250	1.226	1.221	0.051	0.057	0.951	-0.024	0.003	0.057
	ν	2.200	2.220	2.221	0.018	0.024	0.921	0.020	0.001	0.027
	δ	1.150	1.112	1.105	0.054	0.073	0.978	-0.038	0.004	0.067
	β_1	0.250	0.248	0.248	0.024	0.025	0.955	-0.002	0.001	0.024
	β_2	0.350	0.346	0.344	0.048	0.049	0.949	-0.004	0.002	0.048
	β_3	0.150	0.150	0.148	0.049	0.049	0.954	0.000	0.002	0.049
40% Censoring										
GH-BXII	η	1.250	1.241	1.236	0.070	0.072	0.956	-0.009	0.005	0.071
	ν	2.200	2.210	2.211	0.024	0.026	0.948	0.010	0.001	0.026
	δ	1.150	1.132	1.127	0.094	0.106	0.973	-0.018	0.009	0.096
	β_{11}	0.250	0.248	0.248	0.029	0.030	0.964	-0.002	0.001	0.029
	β_{12}	0.350	0.344	0.344	0.059	0.060	0.952	-0.006	0.003	0.059
	β_{13}	0.150	0.150	0.149	0.060	0.059	0.947	0.000	0.004	0.060
	β_{21}	0.000	0.000	0.001	0.020	0.020	0.947	0.000	0.000	0.020
	β_{22}	0.000	0.002	0.001	0.039	0.040	0.953	0.002	0.002	0.039
	β_{23}	0.000	0.000	0.000	0.039	0.039	0.952	0.000	0.002	0.039
AH-BXII	η	1.250	1.241	1.238	0.066	0.067	0.949	-0.009	0.004	0.067
	ν	2.200	2.210	2.210	0.025	0.026	0.938	0.010	0.001	0.027
	δ	1.150	1.132	1.129	0.075	0.087	0.972	-0.018	0.006	0.077
	β_1	0.250	0.249	0.249	0.027	0.028	0.958	-0.001	0.001	0.027
	β_2	0.350	0.345	0.345	0.054	0.055	0.949	-0.005	0.003	0.054
	β_3	0.150	0.150	0.150	0.055	0.055	0.953	0.000	0.003	0.055

MMLE: mean of the MLEs; Mmle: median of the MLEs; E.SD: empirical standard deviation; M.SE: model-based standard error; MSE: mean-square error; RMSE: root-mean-square error; Coverage: coverage proportions.

Table S21: Proportion of Simulation Replicates with Positive Definite Hessian Matrix for the GH-BXII Fitted Model, by Different Data-Generating Structures (GH-1, PH-1, AFT-1, AND AH-1), Sample Size, and Censoring Level

Sample size	Approximate Censoring rate	Data generating structure-FITTED MODEL	Mean observed censoring rate (%)	Convergence (%)	Positive Definite Hessian (%)
1000	20%	GH1-GH	19.94	99.19	99.39
		PH1-GH	21.12	98.52	98.24
		AFT1-GH	23.34	98.08	97.79
		AH1-GH	21.33	97.82	97.53
	40%	GH1-GH	40.36	98.93	98.73
		PH1-GH	40.61	97.13	96.95
		AFT1-GH	40.38	96.31	96.83
		AH1-GH	40.11	95.83	95.41
5000	20%	GH1-GH	19.33	99.72	99.80
		PH1-GH	20.54	99.43	99.37
		AFT1-GH	22.55	99.11	99.04
		AH1-GH	20.49	99.03	98.83
	40%	GH1-GH	39.79	99.25	99.23
		PH1-GH	40.05	98.26	98.05
		AFT1-GH	39.73	97.63	97.33
		AH1-GH	39.45	97.02	96.70