Bayesian Analysis of Markov Based Logistic Model

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Abstract: In analyzing longitudinal data the correlations between responses obtained from same individual need to be taken into account. Various models can be used to handle such correlations. This article focuses on the application of transition modeling using Bayesian approach for analyzing longitudinal binary data. For Bayesian estimation asymmetric loss functions, such as, linear exponential (LINEX) and modified linear exponential (MLINEX) loss function and Tierney and Kadnae (T.K.) approximation has been used. Comparison is made using Bayes factor and Bayesian approach under LINEX loss function can be suggested to estimate the parameters of transition model.

Keywords Bayesian approach, Bayes Factor (BF), Linear exponential (LINEX), Longitudinal data, Markov model, Modified linear exponential (MLINEX).

1. INTRODUCTION

In longitudinal studies, subjects are followed over a specific period of time and information regarding occurrence of a particular event of interest is collected at each follow-up. These repeated observations of the outcome and the associated risk factors characterize the longitudinal data for the subjects of a certain population. Markov chain is a suitable probability model for longitudinal data in which at a given time, the outcome is a categorical variable. The choice of Markov chain arises because they are often a good approximation to the structure of serially dependent data. The dependence relationship is commonly assumed to be of first order. Korn and Whittemore [1] proposed a model to incorporate role of previous state as a covariate to analyze the probability of occupying the current state. Regier [2] introduced a two state transition matrix for estimating odds ratio. Kalbfleisch and Lawless [3] proposed models for analyzing under a continuous time Markov process. Azzalini [4] examined the influence of time dependent covariate on the marginal distribution of the binary outcome variables in serially correlated data. Muenz and Rubinstein [5] proposed a model for analyzing longitudinal data assuming that the sequence of states follows a binary Markov chain. In their study, Muenz and Rubinstein considered a heterogeneous group of individual followed over time and each individual can be in state 0 or state 1 at each time point. They introduced a discrete time Markov chain for expressing the transition probabilities in terms of function of covariates for a binary sequence of presence or absence of a disease. The Markov chain model is one of the most important and effective model for analyzing repeated categorical data. Muenz and Rubinstein [5] employed a logistic regression model to analyze the transitional probabilities from one state to another. Liu [6] showed the application of Markov chain in time series data. Islam *et al*. [7] applied higher order Markov chains, where estimation and test procedure become quite complex due to the increased order of the model. Sirdari *et al*. [8] proposed the goodness of fit test for higher order binary Markov chain models based on marginal distribution. Dey and Islam [9] applied GEE approach in conditional count model for repeated data. Sirdari and Islam [10] employed higher order binary Markov chain model using Health and Retirement Study (HRS) data. Following Muenz and Rubinstein, among the more recent works, noteworthy are Islam and Chowdhury [11, 12], Islam *et al*. [13, 14] and Chowdhury *et al*. [15]. All of them applied classical approach for decision-making.

In some cases, nature of the parameter is random. In that situation classical approaches cannot be applied. Bayesian approach helps us to deal such a situation. Bayesian estimation is extending rapidly in many areas. Hanson *et al*. [16] proposed an informative g-prior for logistic regression. Bayesian approach was applied in many distributions, also applied in Markov model by Markov Chain Monte Carlo (MCMC) algorithm. Noorian and Ganjali [17] was applied Bayesian analysis of transitional model for longitudinal ordinal response data, but in their study, they applied Markov Chain Monte Carlo (MCMC). Acquah [18] also used MCMC applied Bayesian logistic regression for economic data. Although MCMC method is well known and popular but this method is to be solved by programming. There is no theoretical idea

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about the procedure of estimating parameter. To get proper idea of the estimating procedure, this paper, Muenz-Rubinstein model has been estimated theoretically and the numerical findings were obtained using R programming. Mahanta *et al*. [19] applied Bayesian approach under squared error loss function were then compared method of maximum likelihood approach in Muenz-Rubinstein model. Mahanta and Biswas [20] employed Bayesian approach in Azzalini model. In Bayesian approach, loss function is the most important ingredient. Mahanta *et al*. [19] applied Bayesian approach under squared error loss function to estimate the parameters of Muenz-Rubinstein model. Squared error loss function is the symmetric loss function, but most of the practical cases loss is asymmetric in nature. In that situation asymmetric loss function is applied. This paper has been employed Bayesian approach under LINEX and MLINEX loss functions to estimate the parameters of the Muenz-Rubinstein.

2. MODEL

The transition matrix of a two states discrete time-binary sequence Markov chain. The transition matrix is as

$$P = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix}$$

where, P is the transition probability matrix, p_{00} denotes the transition probability from state 0 to 0 and p_{10} is the transition probability from state 1 to 0. At each time point, a vector of length two contains the probability of outcome of interest and its complement.

Muenz and Rubinstein proposed model the transition probabilities p_{00} and p_{10} by logistic regressions.

Where,

$$p_{00} = \frac{\exp(\beta' X)}{1+\exp(\beta' X)} \tag{1}$$

and

$$p_{10} = \frac{\exp(\alpha' X)}{1+\exp(\alpha' X)} \tag{2}$$

The vector X contains covariates and for the q^{th} person in the study is $X_q = (1, X_{q1}, ... X_{qp})$. There are two logistic regressions, one having parameter vector $\beta = (\beta_0, ..., \beta_p)'$ and the other having parameter vector $\alpha = (\alpha_0, ..., \alpha_p)$. Large positive (negative) values of $\beta' X$ and $\alpha' X$ yield large (small) transition probabilities. The above transition probabilities are follows multinomial distribution, for 0 to 0 transitions the joint distribution of the above Markov model is

$$\begin{aligned} f(x/\beta) &= \prod_{i=0}^{n}\left[\{p_{00}\}^{n_{00i}}\{1-p_{00}\}^{n_{01i}}\right] \\ &= \prod_{i=0}^{n}\left[\left\{\frac{\exp(\beta' X)}{1+\exp(\beta' X)}\right\}^{n_{00i}}\left\{\frac{1}{1+\exp(\beta' X)}\right\}^{n_{01i}}\right] \end{aligned} \tag{3}$$

where, n_{00i} and n_{01i} are the number of transitions.

3. PRIOR AND POSTERIOR DISTRIBUTION

Selection of a prior distribution is an important part in Bayesian approach. When proper information is unavailable, then non-informative has been used an extensive tradition in statistics. Mahanta *et al*. [19] applied non-informative prior is Jeffrey's prior along with the uniform prior and defined as $g(\beta) = I$. Where, I represent unit vector and $g(\beta)$ for prior density.

Then the posterior distribution of β for the given sample is

$$f(\beta/X) = \frac{\prod_{i=0}^{n}\left[\left\{\frac{\exp(\beta' X)}{1+\exp(\beta' X)}\right\}^{n_{00i}}\left\{\frac{1}{1+\exp(\beta' X)}\right\}^{n_{01i}}\right]}{\int_{-\infty}^{\infty}\prod_{i=0}^{n}\left[\left\{\frac{\exp(\beta' X)}{1+\exp(\beta' X)}\right\}^{n_{00i}}\left\{\frac{1}{1+\exp(\beta' X)}\right\}^{n_{01i}}\right]d\beta} \tag{4}$$

4. LOSS FUNCTION AND POSTERIOR RISK FUNCTION

Loss function is the important ingredients for Bayesian approach. Linear exponential (LINEX) and modified linear exponential (MLINEX) loss functions have been applied and by the help of posterior risk comparison is made.

4.1. Bayes Estimator under LINEX Loss Function

Let us consider the following LINEX [21] loss function of the form

$$L(\hat{\beta};\beta) = k\left[e^{c(\hat{\beta}-\beta)} - c(\hat{\beta}-\beta) - 1\right] \; ; k > 0, c \neq 0 .$$

We know, Bayes estimator of the parameter β under LINEX [21] loss function is

$$\hat{\beta}_{BL} = -\frac{1}{c}\log E_{\beta}\left(e^{-c\beta}\right). \tag{5}$$

Therefore,

$$E_{\beta}\left(e^{-c\beta}\right) = \frac{\int e^{-c\beta}\prod_{i=0}^{n}\left[\{p_{00}\}^{n_{00i}}\{1-p_{00}\}^{n_{01i}}\right]d\beta}{\int\prod_{i=0}^{n}\left[\{p_{00}\}^{n_{00i}}\{1-p_{00}\}^{n_{01i}}\right]d\beta}$$

The above Bayesian integral appeared in the computation cannot be solvable for this type of model. Tierney and Kadnae [22] approximation have been used to estimate the value of the integral. Tierney and Kadnae [22] proposed that if the form of the integral is

$$I(X) = E\left(u(\beta)/X\right) = \frac{\int u(\beta)e^{L_0(\beta)+p(\beta)}d\beta}{\int e^{L_0(\beta)+p(\beta)}d\beta}$$

where, $u(\beta)$ is the functional form of the expected value with respect to posterior density, L_0 is the log-likelihood and $p(\beta)$ is the log of prior.

Then it can be approximately evaluated as

$$I(X) = \frac{\hat{\sigma}^{*}}{\hat{\sigma}}\exp\left[n\left\{\zeta^{*}\left(\hat{\beta}^{*}\right) - \zeta\left(\hat{\beta}\right)\right\}\right] \tag{6}$$

where,

$$\zeta^{*}\left(\hat{\beta}^{*}\right) = \frac{\log u(\beta)}{n} + \zeta(\beta)$$

$$\begin{aligned}\zeta^{*}(\beta^{*}) &= \log u(\beta) + \zeta(\beta)\\ &= -c\beta^{*} + \sum_{i=1}^{n}\left[n_{00i}\beta^{*}X_i - (n_{00i}+n_{01i})\log\left\{1+\exp\left(\beta^{*}X_i\right)\right\}\right]\end{aligned} \tag{7}$$

$\hat{\beta}^{*}$ maximizes $\zeta^{*}(\beta^{*})$ and β is the posterior mode and therefore maximizes $\zeta(\beta)$ and

$$\hat{\sigma}^{-2} = \left(-\frac{\delta^2\zeta(\beta)}{\delta\beta^2}\right)_{\beta=\hat{\beta}}, \quad \hat{\sigma}^{*-2} = \left(-\frac{\delta^2\zeta^{*}(\beta^{*})}{\delta\beta^{*2}}\right)_{\beta^{*}=\hat{\beta}^{*}}$$

where,

$$\zeta(\beta) = L_0 = \sum_{i=1}^{n}\left[n_{00i}\beta X_i - (n_{00i}+n_{01i})\log\left\{1+\exp(\beta X_i)\right\}\right]. \tag{8}$$

differentiate equation (8) successively with respect to β

$$\frac{\partial\zeta(\beta)}{\partial\beta} = \sum_{i=1}^{n}\left[n_{00i}X_i - (n_{00i}+n_{01i})\frac{\exp(\beta' X_i)X_i}{1+\exp(\beta' X_i)}\right] \tag{9}$$

again,

$$\frac{\partial^2\zeta(\beta)}{\partial\beta^2} = -\sum_{i=1}^{n}(n_{00i}+n_{01i})X_i^2 p_{00}p_{01} \tag{10}$$

and,

$$\hat{\sigma} = \left[\sum_{i=1}^{n}(n_{00i}+n_{01i})X_i^2\hat{p}_{00}\hat{p}_{01}\right]^{-\frac{1}{2}}.$$

Equation (9) and (10) represent score vector and information matrix respectively and using above two equations likelihood estimator of parameter β have been estimated. Again, for estimating the parameter β^{*}, we differentiate equation (7) both sides with respect to β^{*} successively.

$$\frac{\partial\zeta^{*}(\beta^{*})}{\partial\beta^{*}} = -c + \sum_{i=1}^{n}\left[n_{00i}X_i - (n_{00i}+n_{01i})\frac{\exp(\beta'^{*}X_i)X_i}{1+\exp(\beta'^{*}X_i)}\right] \tag{11}$$

again,

$$\frac{\partial^2\zeta^{*}(\beta^{*})}{\partial\beta^{*2}} = -\sum_{i=1}^{n}(n_{00i}+n_{01i})X_i^2 p_{00}^{*}p_{01}^{*} \tag{12}$$

and,

$$\hat{\sigma}^{*} = \left[\sum_{i=1}^{n}(n_{00i}+n_{01i})X_i^2\hat{p}_{00}^{*}\hat{p}_{01}^{*}\right]^{-\frac{1}{2}}.$$

Likelihood estimator of β^{*} can be estimated by using equation (11) and (12).

Therefore, the Bayes estimator under LINEX loss function is

$$\hat{\beta}_{BL} = \hat{\beta}^{*} + \frac{1}{2c}\log\left[\frac{\sum_{i=1}^{n}(n_{00i}+n_{01i})X_i^2\hat{p}_{00}^{*}\hat{p}_{01}^{*}}{\sum_{i=1}^{n}(n_{00i}+n_{01i})X_i^2\hat{p}_{00}\hat{p}_{01}}\right] - \frac{1}{c}D \tag{13}$$

where,

$$D = \sum_{i=1}^{n}\left\{n_{00i}X_i\left(\hat{\beta}^{*} - \hat{\beta}\right) + (n_{00i}+n_{01i})\log\frac{\hat{p}_{01}^{*}}{\hat{p}_{01}}\right\},$$

$\hat{\beta}_{BL}$ represent Bayes estimator under LINEX loss function, c is the shape parameter of the loss function, $\hat{\beta}$ and $\hat{\beta}^*$ are the maximum likelihood estimates [5] of β and β^* respectively.

4.2. Posterior Risk Function under LINEX Loss Function

Posterior risk under LINEX loss function is obtained by

$$R_p(\beta) = \int \left[e^{c\left(\hat{\beta}-\beta\right)} - c\left(\hat{\beta}-\beta\right) - 1 \right] f(\beta / X)\, d\beta$$
$$= e^{\hat{\beta}_{BL}} \int e^{-c\beta} f(\beta / X)\, d\beta - c\,\hat{\beta}_{BL} + c\,\hat{\beta}_{BSE} - 1 \tag{14}$$

where, $R_p(\beta)$ represent the posterior risk and $\hat{\beta}_{BSE}$ is the Bayes estimator under squared error loss function

4.3. Bayes Estimator under MLINEX Loss Function

MLINEX [23] loss function is defined as

$$L\left(\hat{\beta};\beta\right) = k\left[\left(\frac{\hat{\beta}}{\beta}\right)^c - c\log\left(\frac{\hat{\beta}}{\beta}\right) - 1\right]; c \neq 0, k > 0$$

For MLINEX loss function Bayes estimator [23] of parameter β is

$$\hat{\beta}_{BML} = \left[E_\beta\left(\beta^{-c} / x\right)\right]^{-\frac{1}{c}} \tag{15}$$

Functional form under MLINEX loss function of β is $u(\beta) = \beta^{-c}$.

Therefore,

$$\zeta^*(\beta^*) = \log u(\beta) + \zeta(\beta)$$
$$= -c\log\beta^* + \sum_{i=1}^{n}\left[n_{00i}\beta^* X_i - (n_{00i} + n_{01i})\log\left\{1 + \exp(\beta^* X_i)\right\}\right] \tag{16}$$

To estimate the parameter β^*, differentiate both sides successively in equation (16) with respect to β^*.

We have,

$$\frac{\partial\zeta^*(\beta^*)}{\partial\beta} = -\frac{c}{\beta^*} + \sum_{i=1}^{n}\left[n_{00i}X_i - \left(n_{00i} + n_{01i}\right)\frac{\exp(\beta'^* X_i)X_i}{1 + \exp\left(\beta'^* X_i\right)}\right] = 0$$

again,

$$\frac{\partial^2\zeta^*\left(\beta^*\right)}{\partial\beta^2} = \frac{c}{\beta^{*2}} - \sum_{i=1}^{n}\left(n_{00i} + n_{01i}\right)X_i^2 p_{00}^* p_{01}^*$$

and,

$$\hat{\sigma}^* = \left[-\frac{c}{\beta^{*2}} + \sum_{i=1}^{n}\left(n_{00i} + n_{01i}\right)X_i^2\, \hat{p}_{00}^*\, \hat{p}_{01}^*\right]^{-\frac{1}{2}}$$

Therefore, Bayes estimator of β under MLINEX loss function

$$\hat{\beta}_{BML} = \hat{\beta}^* \left[\frac{\left[-\frac{c}{\hat{\beta}^{*2}} + \sum_{i=1}^{n}\left(n_{00i} + n_{01i}\right)X_i^2\, \hat{p}_{00}^*\, \hat{p}_{01}^*\right]^{-\frac{1}{2}}}{\left[\sum_{i=1}^{n}\left(n_{00i} + n_{01i}\right)X_i^2\, \hat{p}_{00}\, \hat{p}_{01}\right]^{-\frac{1}{2}}} \times D\right]^{-1/c} \tag{17}$$

where, $\hat{\beta}$ and $\hat{\beta}^*$ are the maximum likelihood estimates [5] of β and β^* respectively.

4.4. Posterior Risk Function under MLINEX Loss Function

Posterior risk is the expectation of loss function with respect to posterior density

$$R_p(\beta) = \int\left[\left(\frac{\hat{\beta}}{\beta}\right)^c - c\log\left(\frac{\hat{\beta}}{\beta}\right) - 1\right] f(\beta / X)\, d\beta$$
$$= \hat{\beta}_{BML}\int\beta^{-c} f(\beta / X)\, d\beta - c\log\hat{\beta}_{BML} + c\int\log\beta\, f(\beta / X)\, d\beta - 1 \tag{18}$$

The parameters of transition state 1 to 0 can be estimated in similar way.

5. BAYES FACTOR

Considering M_1 and M_2 be the two candidate fitted Markov models of Bayes estimator under LINEX and MLINEX loss function respectively, each specifying a set of data and a prior distribution on this set. Same priori that the models are assigned equal odds. The posterior under model M_k, $k = 1,2$ is given by

$$\left\{posterior \setminus M_k\right\} = \frac{\left\{likelihood \setminus M_k\right\} \times \left\{prior \setminus M_k\right\}}{normalizing\ \ cons\tan t}$$

The normalizing constant, which is simply the integral of $\left\{likelihood \setminus M_k\right\} \times \left\{prior \setminus M_k\right\}$ over the parameter space is called the marginal likelihood under M_k, denoted by $m\left(data \setminus M_k\right)$.

The Bayes factor [24] for comparing model M_1 and M_2 is

$$BF_{12} = \frac{m\left(data \setminus M_1\right)}{m\left(data \setminus M_2\right)}$$

A `large' value of $BF_{12}$ indicates support for $M_1$ relative to $M_2$, and a `small' value (> 0) indicates support for M_2 relative to M_1. Bayes factor consistency refers to the stochastic convergence of BF_{12}, under the true probability distribution, such that $BF_{12} \to \infty$ if M_1 is the best model, $BF_{12} \to 0$ if M_2 is the best model also if $BF_{12} > 1$ then M_1 is preferable than M_2, if $BF_{12} < 1$ but positive then M_2 is preferable than M_1 and if $BF_{12} < 1$ then M_1 and M_2 are same effect.

6. RESULTS

A repeated measures data on Maternal Morbidity in Bangladesh is used. The survey was conducted from November 1992 to December 1993 by the Bangladesh Institute for Research for Promotion of Essential and Reproductive Health and Technologies (BIRPERHT). The subjects comprised of pregnant women with less than 6 months of duration and were followed on regular basis (roughly at an interval of 1 month) throughout the pregnancy. The data of first four consecutive antenatal visits is considered from 548 such women’s information for the analysis. Age at marriage, economic status and any miscarriage are the three highly significant covariates have been applied for fitting the model.

Table 1: Number of Transitions for Pregnancy Complications

Transition Counts			
States	**0**	**1**	**Total**
0	1338	250	1588
1	250	354	604

Bayesian approaches have been applied for estimating the parameters of the Muenz-Rubinstein model considering three highly significant covariates.

Table **2** (shown in the next page) represents that, for c=1, posterior risk of the Bayes estimates under MLINEX loss function is smaller than the posterior risk of the Bayes estimates under LINEX loss function in both transitions 0 to 0 and 1 to 0. Where, any miscarriage and age at marriage are positively and economic status is negatively associated with pregnancy complication in 0 to 0 transition and in 1 to 0 transition, any miscarriage and economic status are positively and age at marriage is negatively associated with pregnancy complication.

In addition, Bayesian approach under LINEX loss function have been found better estimate for the covariate any miscarriage, whereas Bayes estimate under MLINEX loss function have shown better result of the covariates economic status and age at marriage for c=-1 in 0 to 0 transition. Better estimates have been given by Bayes estimate under LINEX loss function for all covariates in transition 1 to 0. Where, Economic status is negatively associated in 0 to 0 transition and all others estimates are positively associated with pregnancy complication.

In both transitions posterior risk of the Bayesian approach under LINEX loss function is smaller than posterior risk of the Bayesian approach under MLINEX loss function when c=3. In transition 1 to 0 any miscarriage is positively associated and remaining all of the covariates is negatively associated with pregnancy complication.

For c=-3, all the Bayes estimates under LINEX loss function have smaller posterior risk than Bayes estimator under MLINEX loss function in both transitions. All the estimates are positively associated with pregnancy complication.

Getting preferable estimate, Bayes factor, $BF_{12} > 1$ for all cases, which indicate that Bayesian approach under LINEX loss function gives better estimate than Bayesian approach under MLINEX loss function.

The performances of estimator between two types of loss function have been shown in the following figures given in the after next page.

The patterns of the above figures are approximately same. From the above figures, it is observed that, Bayes estimator under LINEX loss function have smaller posterior risk than MLINEX loss function except for c=1, where c is the shape parameter of the loss functions.

From the above result we have,

$$R_{BL} < R_{BML} \quad ; for\ c \neq 1$$
$$R_{BML} < R_{BL} \quad ; for\ c = 1$$

Table 2: Estimates of Parameters of Covariate-Dependent Markov Chain Models Using Pregnancy Complication Data

Value of c	Loss Function	LINEX		MLINEX		Bayes Factor (BF_{12})
	0 to 0 Transition					
	Covariate	**Estimate**	**Posterior risk**	**Estimate**	**Posterior risk**	
1	Constant	0.7491	0.9773	1.7553	0.0130	6.8901
	Any miscarriage	-0.8298	0.9904	0.1633	0.0028	
	Economic Status	-1.3018	0.9943	-0.3126	0.0001	
	Age at Marriage	-0.9466	0.9914	0.0456	0.0395	
-1	Constant	2.7326	1.0061	1.7264	1.9770	2.4886
	Any miscarriage	1.1537	0.9930	0.1606	0.9936	
	Economic Status	0.6817	0.9891	-0.3074	0.9220	
	Age at Marriage	1.0369	0.9921	0.0448	0.9751	
3	Constant	1.4103	0.9484	1.7456	0.6942	1.6520
	Any miscarriage	-0.1686	0.9877	0.1624	36.937	
	Economic Status	-0.6406	0.9995	-0.3109	9.3640	
	Age at Marriage	-0.2854	0.9906	0.0453	486.15	
-3	Constant	2.0715	1.0351	1.7360	8.0883	1.3083
	Any miscarriage	0.4925	0.9958	0.1615	1.0409	
	Economic Status	0.0206	0.9840	-0.3092	1.0238	
	Age at Marriage	0.3758	0.9929	0.0451	0.9146	
	1 to 0 Transition					
1	Constant	-1.7145	0.9888	-0.8015	0.0387	3.2723
	Any miscarriage	-0.6325	0.9358	0.3350	0.0034	
	Economic Status	-0.6657	0.9374	0.3001	0.0078	
	Age at Marriage	-1.1393	0.9606	-0.1974	0.0194	
-1	Constant	0.1884	0.9141	-0.7257	0.5342	3.9772
	Any miscarriage	1.2704	0.9671	0.3033	1.0109	
	Economic Status	1.2371	0.9654	0.2717	1.0335	
	Age at Marriage	0.7636	0.9422	-0.1787	1.0870	
3	Constant	-1.0802	1.0636	-0.7754	0.6465	1.4145
	Any miscarriage	0.0018	0.9046	0.3241	8.6310	
	Economic Status	-0.0315	0.9094	0.2903	10.987	
	Age at Marriage	-0.5050	0.9791	-0.1909	26.593	
-3	Constant	-0.4459	0.8393	-0.7501	0.7662	1.4833
	Any miscarriage	0.6361	0.9983	0.3135	1.1995	
	Economic Status	0.6028	0.9934	0.2808	1.2162	
	Age at Marriage	0.1293	0.9238	-0.1847	1.2559	

where, R_{BL} and R_{BML} stands for posterior risk under LINEX and MLINEX loss functions respectively. All the calculations have been performed by R-Language (Version-2.10.0). The program has been shown in appendix.

7. CONCLUSIONS

Longitudinal data are widely used in biological science, social science, engineering etc. This paper has been applied Muenz-Rubinstein model. Bayesian

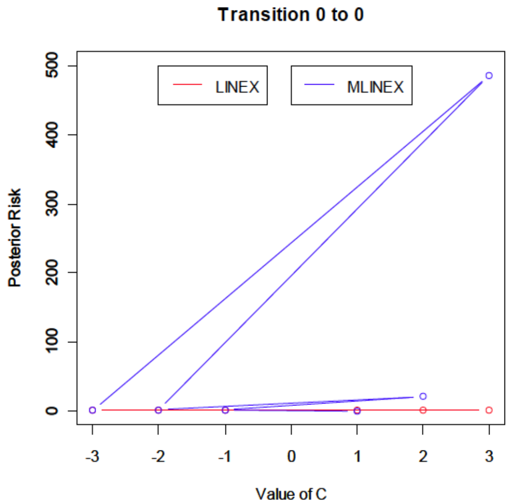

Figure 1: Posterior risk of Bayesian approach under LINEX and MLINEX loss function for covariate Any Miscarriage in 0 to 0 transition.

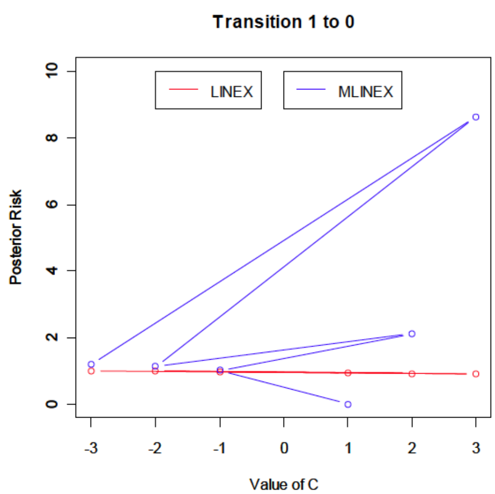

Figure 2: Posterior risk of Bayesian approach under LINEX and MLINEX loss function for covariate Any Miscarriage in 1 to 0 transition.

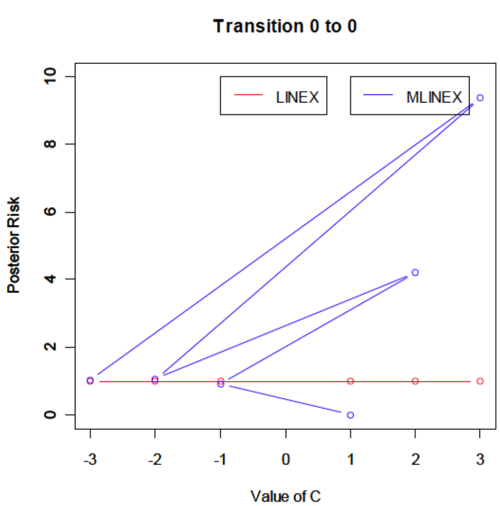

Figure 3: Posterior risk of Bayesian approach under LINEX and MLINEX loss function for covariate Economic Status in 0 to 0 transition.

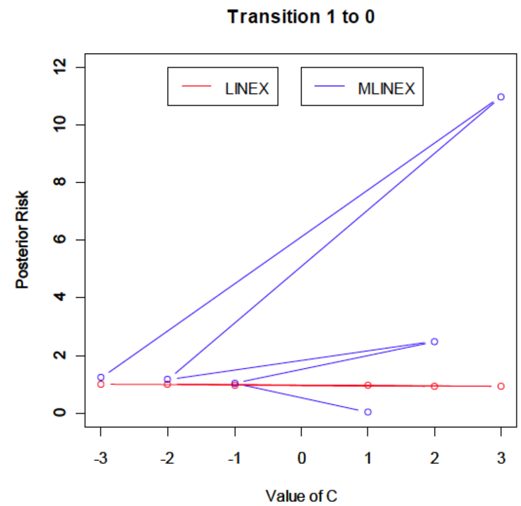

Figure 4: Posterior risk of Bayesian approach under LINEX and MLINEX loss function for covariate Economic Status in 1 to 0 transition.

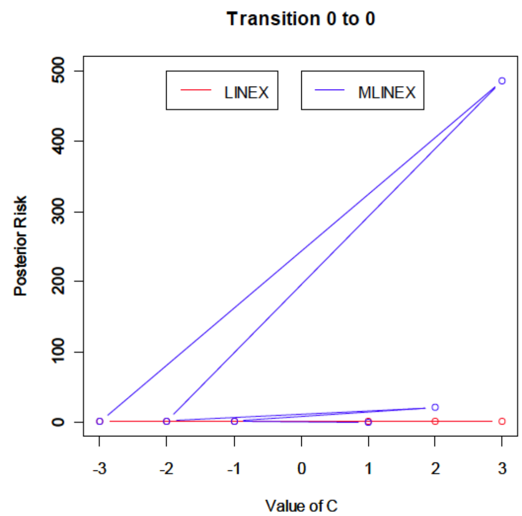

Figure 5: Posterior risk of Bayesian approach under LINEX and MLINEX loss function for covariate Age at Marriage in 0 to 0 transition.

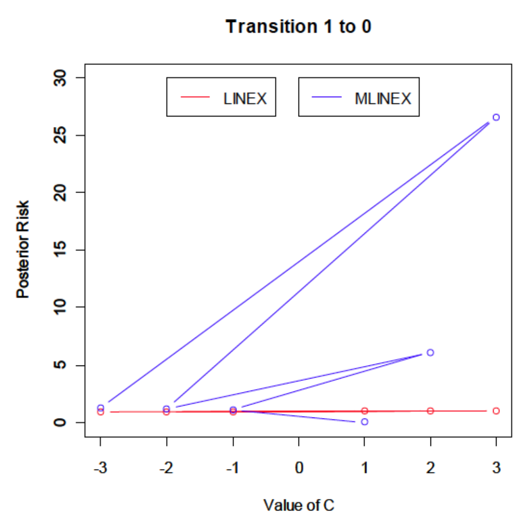

Figure 6: Posterior risk of Bayesian approach under LINEX and MLINEX loss function for covariate Age at Marriage in 1 to 0 transition.

APPENDIX

```
library(foreign,MASS)
data<-as.matrix(read.table("D:/three.txt"))
initial<-c(.1,.1,.1,.1)
c<-(1)
Rub1<-function(data,initial)
{
id<-data[,1]
fup<-data[,2]
A<-data[,3]
B<-data[,4]
C<-data[,5]
count<-0
k<-0
repeat {
b0<-initial[1]
b1<-initial[2]
b2<-initial[3]
b3<-initial[4]
b<-as.vector(c(b0,b1,b2,b3))
infb=matrix(c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0),ncol=4,byrow=T)
infb1=matrix(c(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0),ncol=4,byrow=T)
k<-k+1
x=cbind(1,A,B,C)
g00=exp(x%*%b)/(1+exp(x%*%b))
g01=1/(1+exp(x%*%b))
sb=colSums(x*as.vector(data[,8]-(data[,8]+data[,9])*g00))
sb1=colSums(x*as.vector(data[,8]-(data[,8]+data[,9])*g00))+(1/b)
for(i in 1:ncol(x)){
for(j in 1:ncol(x)){
for(l in 1:ncol(x)){
infb[i,j]=(sum(x[,i]*x[,j]*(as.matrix(data[,8]+data[,9]*(g01)*(g01)))))
}
}
}
b1<-b%*%(t(b))
b2<-(1/b1)
infb1<-b2+infb
Fisinvb=solve(infb)
Fisinvb1=solve(infb1)
count<-count+1
lik<-b+Fisinvb%*%sb
lik1<-b+Fisinvb1%*%sb1
e<-matrix(cbind(lik,lik1),ncol=2)
conv<-abs(initial-lik)
if(conv[1]<=0.001 && conv[2]<=0.001 && conv[3]<=0.001 &&
conv[4]<=0.001)
 break
initial<-lik
}
h00=( exp(x%*%(lik )))/(1+exp(x%*%(lik)))
h01=1/(1+exp(x%*%(lik)))
s01=1/(1+exp(x%*%(lik1)))
s00=(1-s01)
term=exp(sum(as.matrix(data[,8]*x%*%(lik1-
lik)))+sum(as.matrix(data[,8]+data[,9])*log(s01/h01)))
den=(sum(x[,i]*x[,j]*(as.matrix(data[,8]+data[,9]*(h01)*(h01)))))
neo=sum((x[,i]*x[,j]*(as.matrix(data[,8]+data[,9]*(s01)*(s01)))))
neo1<-neo+ (1/(t(lik1)%*%lik1))
bse=lik1%*%((neo1/den)^(-1/2))*term
#calculation of posterior risk
neo2<- neo+ (2/(t(lik1)%*%lik1))
bse2=lik1*lik1%*%((neo2/den)^(-1/2))*term
bbl<-lik1+(1/(2*c))*log(neo/den)-(1/c)*term
cat("The Bayes estimate under LINEX is\n")
print(bbl)
#calculation of posterior risk
xtra<-exp(-c*lik1)*((neo/den)^(-.5))*exp(term)
rbbl<-exp(c*bbl)*xtra-c*bbl+c*bse-1
cat("The Risk of LINEX is\n")
print(abs(rbbl))
bbml<-lik1%*%(((((-c/(t(lik1)%*%lik1))+neo)/den)^(-1/2))*term)^(-1/c)
cat("The Bayes estimate for MLINEX is\n")
print(bbml)
#calculation of posterior risk
alik1<-abs(lik1)
ghj<-(lik1+log(alik1))/(lik1*lik1*log(alik1)*log(alik1))
xtra2<-log(alik1)*(((ghj+neo)/den)^(-.5))*term
xtra1<- (lik1^-c)%*%(((((-c/(t(lik1)%*%lik1))+neo)/den)^(-1/2))*term)
rbbml<-bbml*xtra1-c*log(abs(bbml))+c*xtra2-1
cat("The Risk of MLINEX is\n")
print(abs(rbbml))
#Calculation of Bayes factor
bb00=( exp(x%*%(bbl)))/(1+exp(x%*%(bbl)))
bb01=1/(1+exp(x%*%(bbl)))
sbc=colSums(x*as.vector(data[,8]))%*%bbl
sbd=colSums(as.vector(data[,8]+data[,9])*log(1/bb01))
sbf=sbc-sbd
bm00=( exp(x%*%(bbml)))/(1+exp(x%*%(bbml)))
bm01=1/(1+exp(x%*%(bbml)))
sbmc=colSums(x*as.vector(data[,8]))%*%bbml
sbmd=colSums(as.vector(data[,8]+data[,9])*log(1/bm01))
sbmf=sbmc-sbmd
bfactor=sbf/sbmf
cat("Bayes factor of LINEX with respect to MLINEX\n")
print(bfactor)
}
Rub1(data, initial)
```

approach under LINEX and MLINEX loss functions have been used to estimate the parameters of the model and their comparison is made using Bayes factor for different values of c. This paper reveals that Bayes estimators under LINEX loss function have shown better estimate. That is Bayesian approach under LINEX loss function can be suggested to estimate the parameter of Muenz-Rubinstein model and predict appropriate result about longitudinal data.

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