

Common Stochastic Volatility in International Real Estate Market

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Abstract: This study examined the real estate markets of Europe, North America, and Asia using daily continental real estate indices. It applied a multivariate stochastic volatility model to analyze the behavior of volatility trends in these markets. The results showed comovements in volatilities, especially between Europe and North America, as indicated by high degrees of correlation of their respective stochastic trend components. However, the impact of this common trend varies in these markets, especially for the early period of the sample. For the later period of the sample, the derived volatility trend indicated volatility convergence among them. It might imply that the role of emerging market such as Asia in diversifying real estate investment risk was not as significant as showed in early studies and is diminishing overtime.

Keywords: Real Estate Investment, Volatility, Stochastic Variance.

INTRODUCTION

In recent decades, international real estate investment increasingly drew the attention of portfolio managers who were looking for new diversification opportunities. The stocks of real estate investment trusts (REITs) and other publicly traded real estate companies made the access to real estate investment much easier compared to acquiring real estate properties in an unknown territory thousands of miles away. With international real estate component in the portfolio, it is crucial to understand the relationship among markets from different parts of the world. This study examined publicly traded real estate stocks on the international market with a focus on the markets of Europe, North America, and Asia. It analyzed markets systematically rather than modeling them separately. In this way, it is possible to capture the growing impact from globalization, especially the inter-continental integration. Historically, North America and Europe with well-developed real estate equity markets were market leaders on the world. Recently, Asia is catching up with booming housing market, thirst for new capital investment, and world-class financial centers. Instead sampling major countries as the representatives of their continents, this study adopted a continental real estate index to measure market behavior. It break the country borders and weighted all major real estate companies of a continent into one index which in turn provided the bird's-eye view for a continental market. Return and

risk are double blades of the sword of investment. It is understandable that many people are focusing on the return of their investments. However, the value of investments is not always going up. Real estate investment has no exception. The volatile market situation brings uncertainty for investments. It is especially true if you put your money into a foreign land that even does not belong to your continent.

In this study, we focused on the risky side of international real estate investment by analyzing the dynamics of volatility. It helped to answer several of the following questions: Are all continental markets move together? If yes, are they moving in the same direction? Are they having the same magnitude of movement? Are these movements caused by the same factor? If not, which continental market is more likely to be the trigger of a world-wide market shakeup? The findings could add additional information for the question interests international investors such as "Could the diversification in geographical areas continue to bring the diversification in risk?" Facing recent volatility caused by financial crisis, the results could be important to understand the magnitude of this crisis spreading cross the globe in terms of real estate market movement.

This study utilized a multivariate stochastic variance model to detect the behavior of volatility across continents. This methodology has several merits compared to other prevailing models e.g. multivariate generalized autoregressive conditional heteroskedasticity (GARCH), in volatility analysis. In a multivariate framework, it provides a parsimonious estimation by including much less variables. As other structural time series models, it does not need to impose restrict assumptions such as normality on the

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error term. Finally, this model is able to estimate volatility movements in different markets instantaneously rather than using lagged information.

LITERATURE REVIEW

Past studies showed real estate market integration within its own continent. Eichholtz *et al.* (1998) is one of the few studies investigating the continental factors for major continents including North America, Europe, and Asia-pacific region. With the information of 80's and early 90's, they found that both Europe and North America markets were driven by their continental factors, but the market in Asia-pacific region presented a more diversified aspect which could provide the benefits of diversification. With time passing by, the markets in Asia followed the footsteps of other continents and became more integrated. Gerlach *et al.* (2006) studied the inter-relationship of real estate markets among Asian countries before and after the 1997 Asian financial crisis using data from 1993 to 2001. It showed that the Asian markets were integrated both before and after the crisis and the crisis did not have much impact on the long-term relationship of real estate markets in that region. For Europe, studies also showed the pattern of market integration over time within the continent. From the same group of researchers, in Lizieri *et al.* (2003), they studied the convergence trend in real estate markets in Europe with a small group of countries and found a less intense degree of integration among markets. In McAllister and Lizieri (2006), they expanded their research scope by including both European Union (EU) and non-EU countries and both Euro zone and non-Euro zone countries. Surprisingly, not only countries that were EU members and Euro zone members had increasing market integration over time, but the non-Euro zone members and non-EU members were also having a growing integration among their markets, which implied that the European continental factor dominated the integration process. Finally, for North America, there is strong consensus among economists about the close economic tie among countries, for example, US and Canada. However, there are few studies exploring the inter-relationship on the real estate markets on this continent. Cunningham and Kolet (2007) studied 137 cities in US and Canada using data over two decades. Their findings showed a high correlation of housing price cycles between these two countries. The above within-continent studies suggested a better diversification result could be achieved through outside-continent real estate investments.

The following studies explored the degree of diversification benefits by investing in real estate worldwide. Throughout the years, researchers kept tracking the inter-relationship among real estate markets across the globe. Asabere *et al.* (1991) studies the different performances of US real estate investment and international real estate investment in the 1980's. They discovered that international real estate investment brought both high return and high risk compared to real estate investment in US. Liu and Mei (1998) also mainly focused on the 80s market and include two years in the 90s. With the help of data for six countries representing different part of the world, they showed that the diversification benefits on international real estate market even with the existence of currency risk. Ling and Naranjo (2002) expanded research scope to both 80s and 90s and focusing on the returns of publicly traded real estate companies from 28 countries. The authors realized the existence of a worldwide systematic risk. Their findings implied a diversification benefits for international real estate investment after controlling for the systematic risk. Wilson and Zurbruegg (2003) selected six countries representing America, Europe, and Asia-pacific region and examined their markets from 1980 to 2000. In addition to the linkage among markets, the results showed that bigger markets had significant influence over smaller markets. Yunus and Swanson (2007) is one of the latest studies for international real estate market that focused on the 21st century. It mainly studied US and Asia-pacific region. Their findings presented both long run and short run diversification benefits in the public property markets from these two regions.

However, there are two missing links in the above studies. First, there is no systematic investigation of inter-continent market relationship using measurement beyond country borders, e.g. continental real estate index. Some studies analyzed continental factor or inter-continent relationship only through sampling individual countries. Second, few studies examined the common behavior of volatility in different real estate markets. There are several studies that focused on market volatility. Liow (2007) explored the volatility aspect of international real estate market through weekly data using an ARMA (1, 1) – GJR – GARCH model. It showed the clustered time-varying volatility of housing markets across countries with high persistence and predictability. The above model cannot capture the information of common movement in volatility. Wilson *et al.* (2007) utilized structural time series approach to

Table 1: Descriptive Statistics of Daily Return and Squared Daily Return

	North America		Europe		Asia	
Mean	0.028	0.759	0.041	0.701	0.025	1.881
Maximum	4.859	28.634	5.304	28.130	11.368	129.240
Minimum	-5.351	0.000	-4.860	0.000	-8.473	0.000
Std. Dev.	0.871	1.812	0.836	1.733	1.372	5.474
Skewness	-0.375	6.938	-0.086	7.197	0.189	10.412
Kurtosis	6.755	73.810	7.145	74.149	9.454	165.559
$\hat{\rho}[1]$	0.137	0.206	0.068	0.244	0.128	0.205
$\hat{\rho}[2]$	0.063	0.173	0.006	0.179	-0.023	0.152
$\hat{\rho}[3]$	-0.001	0.152	0.011	0.142	0.049	0.232
$\hat{\rho}[4]$	-0.023	0.178	-0.043	0.157	-0.014	0.206
$\hat{\rho}[5]$	-0.025	0.135	-0.012	0.154	-0.028	0.194
$Q[12]$	75.040	646.210	24.880	909.630	69.534	794.260

Notes: The tables present the summary statistics of daily return and squared daily returns of the North American, European and Asian real estate market. The parameter $\hat{\rho}[k]$ is the estimate of the sample autocorrelation with its asymptotic standard error of $1/\sqrt{T}$ (T is the sample size) at the k^{th} lag. The statistic $Q[12]$ is the Ljung-Box statistic based on the k^{th} order autocorrelation.

model the common movement on the markets of four countries. They realized the existence of some unifying forces cross international market. However, it did not explicitly study the common behavior of volatility. To circumvent the procedural limitations and contribute to the gap in literature, we applied a multivariate stochastic variance model to analyze the common behavior of volatility on across different continental markets.

DATA AND MODEL SPECIFICATION

This study used daily real estate index series from FTSE Group for North America, Europe, and Asia. These indices are designed to securitize real estate across the globe and include publicly traded companies with relevant real estate activities such as ownership, and trading and development of income-producing real estate. According to real estate sector categories of industrial classification benchmark (ICB), major components of the index belong to the sectors of REITs and real estate holding & development. Companies included in each index are screened by free float restrictions and foreign ownership limits to make sure their eligibility for a specific region. In addition, with their liquidity status tested on a periodic basis, all real estate stocks included are actively traded. It provides a great opportunity to detect real estate market dynamics.

The sample period is from January 1, 1997 to October 15, 2007. We use the daily return index defined as $r_{i,t} = 100 \times (\ln p_{i,t} - \ln p_{i,t-1})$, where $p_{i,t}$ is the value of index i at time t and $r_{i,t}$ is the return of index i at time t . The plot of daily return and squared daily return for each market over the sample period¹ shows that North American and European markets were more volatile in the second half of the sample period while Asian market presented significant volatility in the first half of the sample period. For Asia, it might be due to the financial crisis in late 1990s. Subprime housing crisis might be the cause for recent turbulent situation in North America and Europe. The descriptive statistics in Table 1 reports the general situation in each market. For both the daily return and squared return, Asia had the highest standard deviation followed by North America and Europe. On average, it puts Asia as the most volatile real estate market in the sample period. All three indices exhibited high kurtosis value that implied the clustering of time-varying volatility. In addition, the significant Ljung-Box Q-statistics show a high degree of serial correlation in both daily return and squared return for all three markets.

The analysis of the relationship between common factor volatility and linear combinations in the conditional mean can be traced back to early works such as K-factor GARCH in Engle (1987) and latent

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factor autoregressive conditional heteroskedasticity (ARCH) model in Diebold and Nerlove (1989). Harvey *et al.* (1994) also applied similar methods to identify the existence of common trend volatility in the world currency market. A model that accounts for unobserved components of prices, common factors between volatility trends, and short run and long run adjustments between prices would capture important information of international real estate markets. Developing an effective risk management strategy is important for portfolio managers and investors seeking new investment opportunities. One aspect of that strategy is to be able to hedging against risk in various markets. This relies on a good understanding of the behavior of investment instruments that would help improve portfolio selection.

This necessitates a flexible approach to account for the common factors and short run and long run adjustments. The multivariate stochastic volatility technique models volatility as an unobserved stochastic process (McMillan, 2001). It enables practitioners to circumvent the problem of dimensionality known to many multivariate GARCH procedures. The parsimonious nature of this technique explains its preference over several multivariate GARCH models.

The most common tools used to measure price risks are derived from the family of ARCH models. These models provide a framework to measure volatility (i.e., risk) as a function of time. They have been generalized and expanded to include multivariate forms with constant and time varying conditional correlations. Alternatively, volatility can also be modeled as an unobserved stochastic process embedded in the return series. This falls into the category of stochastic volatility (or variance) models. A simple univariate stochastic volatility model is specified as follows:

$$y_t = \sigma \varepsilon_t \exp(h_t/2), \quad \varepsilon_t \sim N(0,1), \quad t = 1, \dots, T \quad (1)$$

$$h_t = \phi h_{t-1} + \eta_t, \quad |\phi| < 1 \text{ and } \eta_t \sim N(0, \sigma_\eta^2), \quad t = 1, \dots, T \quad (2)$$

Where y_t is the demeaned return series (i.e., $y_t = r_t - m$ with m the mean of the daily return series), σ is a scale factor, h_t is an underlying unobserved volatility, ε_t and η_t are uncorrelated white noise disturbances that drive the stochastic properties of the daily return and its underlying volatility. The

stochastic process governing the evolution of conditional volatility is a discrete time version of the Ornstein-Uhlenbeck process (Harvey *et al.* 1994; Lo and Wang, 1995) whereby the underlying volatility evolves as a first-order autoregressive process (Lo and Wang, 1995; Pindyck, 1999). Equation (1) can be linearized by log transformation of the squared returns to yield the following

$$\log y_t^2 = \log(\sigma^2) + h_t + \log(\varepsilon_t^2), \quad t = 1, \dots, T \quad (3)$$

Equation (3) and (2) combined forms a linear state space model that can be estimated efficiently by quasi-maximum likelihood using the Kalman filter. In structural time series terminology, equation (3) is referred to as the measurement equation and equation (2) as the transition equation. While the linear transformation leads to a non-normal error distribution, the state space with the Kalman filter yields valid estimates of the conditional variance. In practice, the estimated autoregressive parameter is close to one; in this case, specifying the conditional volatility as a random walk would be justified; hence, a correct specification of equation (2) would be $h_t = h_{t-1} + \eta_t$.

Understanding the dynamic relationships between markets is important for an efficient risk management strategy. This requires modeling volatility in a multivariate framework. Thus, we derived a multivariate stochastic volatility model following Harvey *et al.* (1994) and Koopman *et al.* (2000). The model is as follows:

$$\mathbf{Y}_t = \boldsymbol{\omega} + \mathbf{h}_t + \boldsymbol{\xi}_t, \quad \boldsymbol{\xi}_t \sim NID(\mathbf{0}, \mathbf{D}_\xi) \quad (4)$$

$$\mathbf{h}_t = \mathbf{h}_{t-1} + \boldsymbol{\eta}_t, \quad \boldsymbol{\eta}_t \sim NID(\mathbf{0}, \boldsymbol{\Sigma}_\eta) \quad (5)$$

where $\mathbf{Y}_t = \{\log y_{it}^2\}$, $\boldsymbol{\omega} = \{\omega_i\}$ with $\omega_i = \log \sigma_i^2 + E(\log \varepsilon_{it}^2)$, $\mathbf{h}_t = \{h_{it}\}$ is the vector of stochastic volatility with $t = 1, \dots, T$ and $i = 1, \dots, N$. Moreover $\boldsymbol{\xi}_t$ and $\boldsymbol{\eta}_t$ are $N \times 1$ vectors of white noise disturbances that drive the underlying stochastic properties of the return and volatility series. It is important to point out that $\boldsymbol{\eta}_t = \{\eta_{it}\}$ and $\boldsymbol{\xi}_t = \{\xi_{it}\}$ with $t = 1, \dots, T$, $i = 1, \dots, N$, and $\xi_{it} = \log \varepsilon_{it}^2 - E(\log \varepsilon_{it}^2)$ where ε_{it} is the irregular component in the univariate stochastic volatility model. This transformation diagonalizes the variance of $\boldsymbol{\xi}_t$. Each of these stochastic components is assumed to follow a multivariate distribution with $\mathbf{0}$ and variance \mathbf{D}_ξ and $\boldsymbol{\Sigma}_\eta$. The variance-covariance matrix $\boldsymbol{\Sigma}_\eta$ also accounts for the correlation between the volatilities of the series

and assesses the degree to which they move together. A less than full rank Σ_{η} indicates that volatilities in the international real estate market may be driven by a common underlying trend. In this case, the following adjustments are carried out in the specification to account for the presence of linear combinations $\mathbf{h}_t = \Theta \tilde{\mathbf{h}}_t + \tilde{\mathbf{h}}_0$, $\Sigma_{\eta} = \Theta \mathbf{D} \eta \Theta'$ with $\mathbf{D} \eta$, the diagonal matrices with diagonal elements corresponding to the eigenvalues of the volatility trend innovations' variance matrices. Θ is $N \times K$ factor loading matrices with $K \leq N$ and $\Theta = \{\theta_{ij}\}$. The elements θ_{ij} are constrained to zero for all $i > j$ to ensure identification of the system. $\tilde{\mathbf{h}}_t$ is an $N - K \times 1$ vector of common stochastic trends (random walk plus noise). $\tilde{\mathbf{h}}_0$ is an $N \times 1$ vector of constant with the first K elements equal to zero. The factor loading matrices measure the relationship between the stochastic volatility and the K common volatility trends. Reduced ranks are tested following multivariate unobserved component approach in Harvey *et al.* (1994). The test looks at the number of non-zero elements in $\mathbf{D} \eta$, which equals the number of non-zero columns in the variance matrices. This approach is based on factor analysis and is found more reliable than methods based on autoregressive approximations (Harvey, Ruiz, and Shephard, 1994). Autoregressive methods such as the Johansen procedure rely on unit root testing, which may be difficult to ascertain because of low power of test such as the augmented Dickey-Fuller test. Although there are unit roots tests that are more robust than others, the results are generally determined by whether a constant were used, number of lags, and the chosen significance level. The unobserved state variables, variance parameters, and factor loading matrices are simultaneously estimated by quasi-maximum likelihood procedure using the Kalman filtering technique.

EMPIRICAL RESULTS

We initially estimated three univariate stochastic volatility models with their respective variance following a first-order autoregressive process. The results in Table 2 indicate a high degree of volatility persistence in all markets with the estimated autoregressive coefficients and their respective 95% asymptotic confidence intervals are 0.959 [0.937 0.971] for North America, 0.971 [0.949 0.981] for Europe, and 0.983 [0.969 0.989] for Asia. We calculated the half-life decays estimated by $\log(0.5)/\log(\hat{\phi})$ and found that while all volatilities were persistent there were some differences between the three regions: it takes 42 days for an unanticipated shock in the Asian market to lose half of its initial impacts. For the European and North American markets, the half-life decays were 24 and 17 days, respectively. The stochastic process that shapes the temporal evolution of volatility is driven by σ_{η}^2 , the estimates of which were significant at the 95% confidence level.

The degree of persistence in the three markets is an indication that volatility could also be modeled as a random walk process. We tested the random walk hypothesis using a likelihood ratio test based on the results summarized in Tables 2 and 3 and found it to be valid in all three cases. Thus, we proposed a multivariate random walk with noise as the best avenue to investigate volatility comovements in the international real estate markets. The estimation results based on the full sample are summarized in Table 4. The estimated variance-covariance matrix of the volatility stochastic component revealed some interesting feature worth investigating. While volatilities of the North American and Asian market appear to move stochastically, that of Europe did not. A further analysis of the eigenvalues and eigenvectors of variances pertaining to the volatility trend and irregular

Table 2: Univariate Autoregressive Stochastic Volatility Models

Parameter	Label	North America	Europe	Asia
ϕ	Autoregressive Component	0.959 [0.937 0.971]	0.971 [0.949 0.981]	0.983 [0.969 0.989]
σ_{η}^2	Trend Disturbances	0.734 [0.494 1.089]	0.346 [0.198 0.604]	0.388 [0.159 0.951]
σ_{ε}^2	Irregular Disturbances	5.551 [5.242 5.878]	5.327 [5.044 5.626]	5.425 [5.141 5.724]
Log L	Log Likelihood	-2493.80	-2390.00	-2399.43
Q[12]	Autocorrelation	5.771	15.584	17.657

Notes: The table presents the parameter estimates with their respective 95% asymptotic confidence intervals and the log likelihood of each univariate model. The values of Q[12] are the derived Ljung-Box statistic for the residuals; they are asymptotically χ^2 distributed. The derived Q[12] statistics are less than 21.026, the χ^2 [12] at the 5% level; thus indicate no autocorrelation.

Table 3: Univariate Random Walk Stochastic Volatility Models

Parameter	Label	North America	Europe	Asia
σ_{η}^2	Trend Disturbances	0.012 [0.005 0.026]	0.003 [0.001 0.009]	0.002 [0.001 0.006]
σ_{ϵ}^2	Irregular Disturbances	5.717 [5.408 6.044]	5.395 [5.112 5.693]	5.443 [5.161 5.741]
Log L	Log Likelihood	-2504.09	-2393.90	-2401.19
Q[12]	Autocorrelation	20.625	17.596	18.408

Notes: The table presents the parameter estimates with their respective 95% asymptotic confidence intervals and the log likelihood of each univariate model. The values of Q[12] are the derived Ljung-Box statistic for the residuals; they are asymptotically χ^2 distributed. The derived Q[12] statistics are less than 21.026, the χ^2 [12] at the 5% level; thus indicate no autocorrelation.

Table 4: Multivariate Random Walk Stochastic Volatility Models

Parameter	Label	North America	Europe	Asia
σ_{η}^2	Trend Disturbances	0.013 [0.004 0.042]	0.000 [0.000 0.011]	0.002 [0.000 0.007]
σ_{ϵ}^2	Irregular Disturbances	5.720 [5.207 6.284]	5.344 [4.864 5.871]	5.366 [4.903 5.872]
Q[12]	Autocorrelation	17.582	17.372	17.712

Notes: The table presents the parameter estimates with their respective 95% asymptotic confidence intervals and the log likelihood of the multivariate random walk stochastic volatility model. The values of Q[12] are the derived Ljung-Box statistic for the residuals; they are asymptotically χ^2 distributed. The value of log likelihood is -7263.89. The derived Q[12] statistics are less than 21.026, the χ^2 [12] at the 5% level; thus indicate no autocorrelation.

components indicates comovements between the volatilities based on the estimated eigenvalues of the stochastic components. The principal component analysis of the variance-covariance matrix of the stochastic components (Table 5) shows almost no contribution from the Asian index while North American and European indices contribute almost 99% of the total variance pertaining to the trend. The estimated

correlation matrix is $\Omega(\eta) = \begin{bmatrix} 1 & & \\ 0.969 & 1 & \\ 0.831 & 0.841 & 1 \end{bmatrix}$. Thus,

there is indication of volatility comovements between European and North American volatilities illustrated by a high degree of correlation (0.969) between the stochastic components of their respective volatilities. Moreover, a plot of the derived volatilities (Figure 1) shows shifting dynamics in the international real estate market. While volatilities in the European and North American indices were slightly trending up, that of Asia was following a reverse pattern. The three volatilities appear to converge around March 2002. There are numerous underlying economic events happened during the sample period. For example, creation of Euro in December, 1999; Dot-Com bubbles in 2000; the troubled European and US economies between 2000 and 2001; China becoming a member of World Trade Organization (WTO) in 2000; the "911" terrorist attack in 2001; and fully economic recovery, especially

IT industry, after 2002. All of the above might contribute to the evolution of real estate market activities. Then, we divided the data into several subsample periods. The estimated correlation matrices of the underlying stochastic components at different subsample periods illustrate the converging movements of volatilities in the three markets. The estimated correlation matrix is

$\Omega(\eta) = \begin{bmatrix} 1 & & \\ 0.827 & 1 & \\ 0.331 & 0.138 & 1 \end{bmatrix}$ between January 1997 and

December 1999, $\begin{bmatrix} 1 & & \\ 0.986 & 1 & \\ 0.899 & 0.960 & 1 \end{bmatrix}$ between January

2000 and October 2007, $\begin{bmatrix} 1 & & \\ 0.989 & 1 & \\ 0.901 & 0.956 & 1 \end{bmatrix}$ between

January 2001 and October 2007, and $\begin{bmatrix} 1 & & \\ 0.984 & 1 & \\ 0.937 & 0.984 & 1 \end{bmatrix}$ between March 2002 and October

2007. This may imply that the troubled European and US economy between 2000 and 2001 led investors to spread investment risks through portfolio diversification, which include investing outside their traditional geographic areas.

Table 5: Results of Principal Components of the Variance-Covariance Matrix of the Trend Components

Label	North America	Europe	Asia
North America	0.725	-0.384	-0.572
Europe	0.542	-0.194	0.818
Asia	0.425	0.903	-0.068
Eigenvalues	0.024	0.131×10^{-2}	0.028×10^{-2}
Percentage (%)	93.730	5.166	1.107

Notes: The estimates correspond to the eigenvector and eigenvalues of the variance-covariance matrix Σ_{η} of the stochastic components that drive evolution of the conditional variance. The number of nonzero eigenvalues is the rank of the corresponding matrix.

Table 6: Estimated Factor Loadings (Θ_{μ}) and Community Scores of the Volatility Trends

	Unstandardized	Standardized	Rotated	Community Score
North America	-0.617	1.000	0.617	0.380
Europe	-0.658	1.067	0.658	0.433
Asia	-0.432	0.701	0.732	0.187

Notes: The matrix Θ_{μ} measures the loading of each volatility series on the common volatility trend and community is the proportion of the variance of the volatility series contributed by the common trend, and $h_p = (0, -0.097, -0.349)'$ is the estimated vector of constant pertaining to the trend. The restricted log-likelihood value was evaluated at -3723.74.

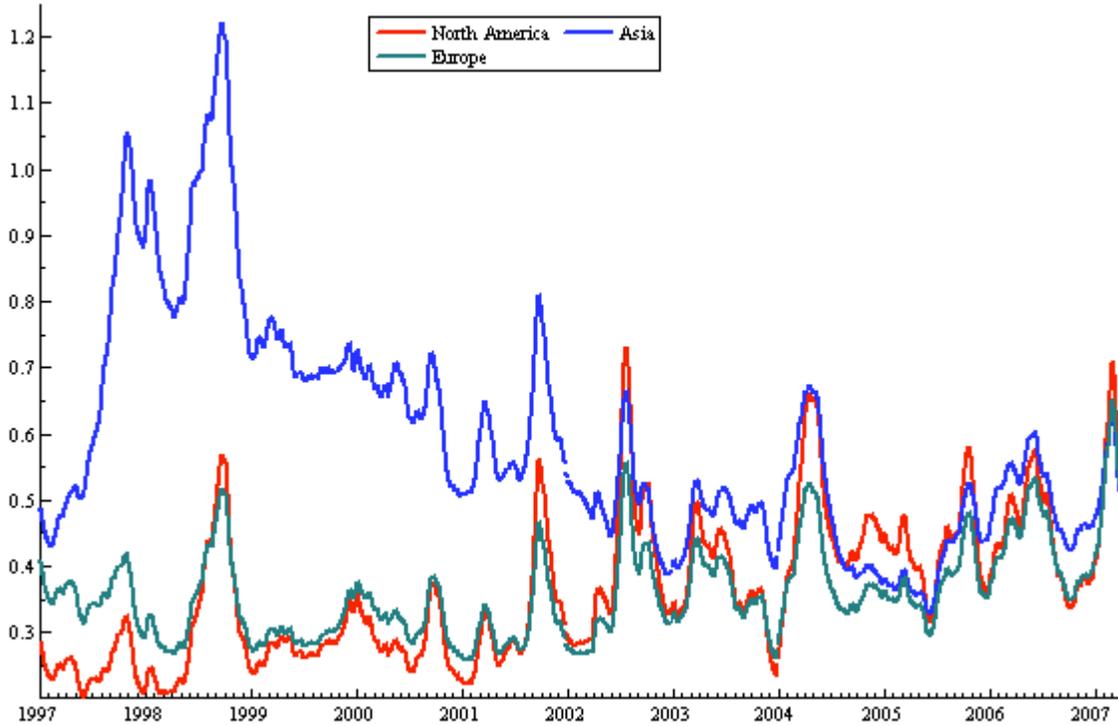


Figure 1: Evolution of Daily Stochastic Volatility of North America, Europe, and Asia Index Return.

Notes: The volatility is measured as $\exp(0.5 \times h_{it})$.

We re-estimated the multivariate random walk using the data between March 2002 and October 2007 and found one common stochastic trend shared by the three markets. The estimated standardized factor loading matrices yielded the following relationships between the three conditional volatilities and the underlying common stochastic volatility using the equation $\mathbf{h}_t = \Theta \tilde{\mathbf{h}}_t + \tilde{\mathbf{h}}_0$.

$$\begin{bmatrix} h_{1t} \\ h_{2t} \\ h_{3t} \end{bmatrix} = \begin{bmatrix} 1 \\ 1.067 \\ 0.701 \end{bmatrix} \tilde{h}_t + \begin{bmatrix} 0 \\ -0.097 \\ -0.349 \end{bmatrix},$$

where h_{1t} , h_{2t} , and h_{3t} refer to the conditional volatilities in the North American, European, and Asian markets, respectively while \tilde{h}_t is the common stochastic volatility trend. The factor loading and their respective 95% asymptotic confidence intervals are 1.067 [0.347 1.786] and 0.701 [0.132 1.269]. The results are summarized in Table 6. The relationships between conditional volatility in each market and the underlying common stochastic volatility trend is further established by using the orthogonal transformations of the factor loading matrix, which were squared to derive the communality score between these markets and the common stochastic volatility trend. We found that the common volatility trend account for 38.0% of the variance of conditional volatility in the North American market, 43.3% of that in the European market, and 18.7% of that in the Asian market.

A long run relationship between the volatilities of the three markets existed between March 2002 and October 2007. Thus the three volatilities are cointegrated with cointegrating vector $(1, -0.937, -1.427)$. The presence of cointegration has important policy implications for investors and portfolio managers. First, it is important to note that this is a recent phenomenon, which was traced in 2000 when economic conditions in the U.S and Europe led more investors to the growing Asian real estate market. Although the common factor affected Asian market in a less extent, it is clear that The North American, European, and Asian markets will continuously adjust to each other and their relative return volatility determines how investor would construct the optimal investment portfolio. The presence of a long-run equilibrium relationship implies that any deviation between the three markets is reestablished in the long run. Furthermore, while Asian market might have been a good source of portfolio diversification for U.S. and European investors in the short run, this might not be the case in recent years. The convergence of volatility trends in the three markets in recent years indicates diminishing diversification gain obtained by having

three indices in one's portfolio for the long run. Looking at the dynamics of these three international markets after 2008 global economic crisis. It showed similarity to the findings from the sample of this study. During and after the global economic crisis, the markets of North America and Europe moved closely. Although Asian markets did not react with the same amplitude, its fast recovery was not as sustainable as expected and went over its peak in recent years. If this trend persists, the slow recovery of North America and Europe markets will meet the adjustment of Asian market.

CONCLUDING REMARKS

The objective of the study was to examine volatility dynamics in the world real estate market. A multivariate stochastic volatility was used to estimate the common stochastic volatility trend between North American, European, and Asian real estate markets. The presence of common factor implies that the three markets are cointegrated, which is indicative of a competitive world real estate market and the existence of the established long-run relationships among them. However, the influence of this common factor varies in different markets. The derived factor loadings and the dynamic interactions between the three markets are findings that could be of great interest to portfolio managers and researchers interested in cross border risk mitigation. Our results indicate diminishing gain in terms of portfolio diversification in recent years due to volatility convergence in the international real estate markets.

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