Social Choice as a Continuous Mapping from $IR^n \rightarrow IR$: A Group Invariance Approach

Lyle Noakes[1,*] and Alex Coram[2]

[1]School of Mathematics and Statistics, The University of Western Australia, 35 Stirling Highway, Crawley WA6009, Australia

[2]School of Social Sciences, The University of Western Australia, 35 Stirling Highway, Crawley WA6009, Australia and School of Social Sciences, The University of Tasmania, Sandy Bay Campus, Hobart, TAS 7001, Australia

Abstract: Social choice is studied by employing a group invariance approach in a way that appears not to have been previously been exploited. This simplifies the problem and the proofs and provides some new insights into the foundations of impossibility results.

Keywords: Aggregating preferences, welfare function, group actions.

1. INTRODUCTION

Social choice theory is concerned with the problem of aggregating individuals' reports of their preferences over alternative states of the world into a ranking of these states. This raises some very simple questions. Under what conditions is such a ranking possible? What are its characteristics when it exists? Despite their simplicity, these questions present some difficulty and are now addressed from various angles by an enormous literature, much of which has been thoroughly surveyed in the Handbook of Mathematical Economics and the Handbook of Social Choice and Welfare.[1] Even given this body of work, there continue to be gains in understanding from employing new techniques to simplify the problem and give fresh perspectives. Some attempt is made to do this here by using a kind of group invariance that, to the best of our knowledge, has not previously been used.

The main feature of the framework we use to interpret the problem is to treat the mechanism that aggregates reports as a continuous mapping from the space of preferences, thought of as a subset of IR^n, to the real numbers IR. In this case the question becomes, under what conditions can this mapping be carried out in a consistent manner?

*Address correspondence to this author at the School of Mathematics and Statistics, The University of Western Australia, 35 Stirling Highway, Crawley WA6009, Australia; Tel: (+618) 64883358; Fax: (+618) 64881028;
E-mail: Lyle.Noakes@uwa.edu.au

Interpreting preferences as points in IR^n is a fairly natural idea and has already been widely used in choice theory. Sen A, 1974 (Sen A, 1986 pp. 1111-28), for example, treat reports as utilities and allow for the fact that these are not uniquely specified by defining the family of functions that carry equivalent information.[2] Roemer also presents choice problems in this framework (Roemer J, 1996).

This idea is developed in this paper by focusing directly on the mathematical structure of the problem in a way that might facilitate further analysis and by exploiting the properties of continuously differentiable functions.[3] For example there is an equivalent in (Roemer J, 1996) to the theorem presented in the last section of our paper. In our paper, however, this theorem is simply an immediate corollary of developments in a different analytical framework.

We set out the paper as follows. In the next section the interpretation of the choice problem is explained. In the following sections we prove the main theorem and its corollaries for welfare functions under different conditions.

2. THE SOCIAL ORDERING PROBLEM

2.1. The Problem

The possible states of the world are given by a topological space V and these are to be ranked by a welfare function, φ, that assigns an ordering based on information on the preferences of some finite number off individuals, n, over every element in V. Information

[1]Sen's earlier survey (Sen A. 1986) remains a classic. The Handbook of Social Choice and Welfare contains more recent surveys including those that deal with economic domains (d' Aspermont, C., and Gevers, L., 2002) (Le Breton, L. and Weymark, J. 2002).

[2]Recent developments in work that follows Sen's approach are covered in (d' Aspermont, C., and Gevers, L., 2002).
[3]For an impossibility theorem based on a different version of this idea see (Coram and Noakes, 2010).

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on preferences is given by any member of a set of continuous functions that contain equivalent information under some condition π. This set is $\xi_\pi = \{\bar{x}, \bar{y}, \ldots\}$ where for each $\bar{x} \in \xi_\pi$

$$\bar{x} : V \to X \subset IR^n$$

and the report of individual i on state of affairs $v \in V$ is taken to be the projection of $\bar{x}(v) = x \in IR^n$ onto the ith axis. If i prefers v to v' then $x_i(v) > x_i(v')$. It is natural from a mathematical viewpoint to think of ranking these reports by a mapping into the reals.[4] In this case the welfare function can be defined as:

$$\varphi : IR^n \to IR$$

If the information conveyed about preferences over states of the world does not have a unique representation ξ_π will contain more than one member. For example, if a report only contains information about rankings, ξ_π will contain all $\bar{x}$ and $\bar{y}$ etc with the property that whenever v is preferred by i to v' we have $x_i(v) > x_i(v')$ and $y_i(v) > y_i(v')$ for all i and $v \in V$.[5]

It simplifies matters to concentrate on the space X rather than the functions in ξ_π and to deal with equivalent information by constructing a family of transformations that can be used to map x to all y for all $\bar{x}, \bar{y} \in \xi_\pi$. It is assumed that a permissible transformation operates on each individual's reports independently of its operation on the reports of other individuals. This means a transformation f will be required to operate componentwise on each element of x to give $f(x) = (f_1(x_1), f_2(x_2), \ldots, f_n(x_n))$.

It follows from the nature of the problem that permissible transformations should form a group: (i) if f and g are transformations then fg is a transformation; (ii) the transformation $f(x) = ex$ where e is the identity is always allowed; (iii) if f is permissible so is g where $fg = gf = e$ the identity element.

In the present paper we restrict attention to some cases of actions by groups that are continuous and path-connected. As noted by a referee, compared to the standard arrovian or senian analysis, this is a stringent condition to impose on the social welfare function. However we make some rather precise statements within this limited scope.

These conditions are summarized in the following axiom:

[A1]: A transformation is permissible under information condition π if it is an element of the continuous path-connected group G_π where

$$G_\pi = \{f : x \mapsto f(x) \ \ where \ \ x \ \ and \ \ f(x) \ \ convey \ \ the \ \ same \ \ information \ \ under \ \ condition \ \ \pi\}$$

Since the content of all permissible transformations under the given information conditions is the same, a necessary condition for a welfare function to be acceptable is:

[A2] φ is acceptable under information condition π if and only if

$$\varphi(x) > \varphi(y) \Rightarrow \varphi(f(x)) > \varphi(f(y)) \tag{1}$$

2.2. Additional Conditions on the Welfare Function

A welfare function is also required to satisfy the following conditions:

[A3] φ is continuous and everywhere differentiable.

[A4] φ is not constant anywhere or dictatorial.

A welfare function is dictatorial if there exists an agent i and some acceptable monotonic function $\sigma : IR \to IR$ such that for all $x \in X$ we have $\varphi(x) = \sigma(x_i)$. It is obvious that any constant or dictatorial function always trivially satisfies equation (order).

2.3. Equality Conditions on the Welfare Function

The inequality conditions in equation (1) can be replaced by equality conditions as follows

Theorem 1. Suppose φ satisfies Conditions $[A2 - A4]$ for all permissible $f \in G_\pi$. Then $\varphi(x) > \varphi(y) \Rightarrow \varphi(f(x)) > \varphi(f(y))$ for all x, y if and only if $\varphi(x) = \varphi(y) \Rightarrow \varphi(f(x)) = \varphi(f(y))$.

Proof of Theorem 1. For the if part suppose $\varphi(x) = \varphi(y) \Rightarrow \varphi(f(x)) = \varphi(f(y))$ and assume $\varphi(x) > \varphi(y)$ and $\varphi(f(x)) < \varphi(f(y))$. Since G_π is path

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[4]We simply shortcut the literature on the relation between the Bergson-Samuelson welfare function and choice functionals. See (Roemer J, 1996, p. 29) for references.

[5]The information content of reports is discussed in detail by Sen with reference to the literature on utility functions (Sen A, 1986).

connected there is a continuous function $\gamma : [0,1] \to G_\pi$ with $\gamma(0) = e$ and $\gamma(1) = f$. Let $q(t) = \varphi(\gamma(t)(x)) - \varphi(\gamma(t)(y)))$. Since φ and γ are continuous q is a continuous mapping $q : [0,1] \to IR$ with $q(0) > 0$ and $q(1) < 0$. It follows from the intermediate value theorem that there is some $t = s \in [0,1]$ with $q(s) = 0$. This means that $\varphi(\gamma(s)(x)) = \varphi(\gamma(s)(y))$. It now follows that $\varphi(f(x)) = \varphi(f(y))$ for all $f \in G_\pi$. This contradicts the assumption $\varphi(x) > \varphi(y)$. Only if is immediate.

2.4. Continuous Parameter Transformations

Of particular interest is the case where the elements of the group can be identified with continuously varying parameters $a = (a_1, a_2, \ldots, a_n)$. Indeed we are most interested in situations where G_π is a *smooth manifold* in the sense of the following definition.

Definition 1. A function from an open subset of IR^m to IR^n is said to be *smooth* when it has continuous partial derivatives of all orders. A *diffeomorphism* is a smooth map with a smooth inverse. A subset M of IR^k is a *smooth manifold* of dimension when it has an open cover by sets diffeomorphic to open subsets of IR^m.

When G_π and X are smooth manifolds, a smooth *left action* of G_π on X is defined to be a smooth function $(a,x) \in G_\pi \times X \mapsto a \circ x \in X$ with the property that $a \circ (b \circ x) = (ab) \circ x$ where ab denotes the product of a,b in the group G_π. In practice G_π will be a group of transformations from X to itself, with the group operation corresponding to compositions of transformations. We then replace $[A2]$ with:

[A5] φ is acceptable for the continuous group G_π with a smooth left action on the manifold $X \subset IR^n$ if there exists a smooth function $\psi : G_\pi \times IR \to IR$ such that for all $a \in G_\pi$ and $x \in X$

$$\varphi(a \circ x) = \psi(a, \varphi(x)) \tag{2}$$

Definition 2. A welfare function that satisfies $[A3-5]$ under a permissible group of transformation G_π will simply be called G_π -acceptable.

In what follows we place the emphasis on the mathematical structure of choice under the group operations and only make brief reference to the information conditions they imply. This loses a little in connection with the literature but gives gains in terms of clarity.[6] For generality we let all group transformations operate on a p dimensional subspace of X where $1 < p \le n$. We start with the translations.

3. TRANSLATIONS

Suppose the information condition allows reports of individuals to be translated by adding a constant, not necessarily the same for each individual. In this case the difference $x_i(v) - x_i(v')$ is retained and there is enough information to make interpersonal comparisons between reports on states of the world (d' Aspermont and Gevers 2002, p. 60, Roemer, J. 1996, p.18). This transformation will be trivial if the constant is the group identity $e = (0,0,\ldots,0)$. The following theorem establishes the conditions that must be met by every acceptable function and provides the foundation for further results.

Let G_t be a continuous group given by a real p dimensional vector space where $n \ge p > 1$ that acts non-trivially on the first p reports as follows. Define a left action of G_t on X by

$$a \circ x := (a_1 + x_1, a_2 + x_2, \ldots, a_p + x_p, x_{p+1}, \ldots x_n)$$

where $a \in G_t$ and $x \in X$. As usual, the Euclidean inner product $\langle , \rangle$ on IR^n is defined by $\langle x, y \rangle = \sum_{i=1}^n x_i y_i$ where x_i and y_i are the components of the n -tuples x and y in IR^n. Take W to be the orthogonal complement $W := G_t^\perp$ of G_t with respect to $\langle , \rangle$ on IR^n. Then W consists of all n -tuples of the form

$$(0,0,\ldots,0,w_1,w_2,\ldots w_{n-p}) \in IR^n.$$

As in Figure **1**, W can be identified with IR^{n-p} and any $x \in X$ is uniquely of the form $h(x) + w(x)$ where $h(x) := (x_1, x_2, \ldots, x_p) \in G_t$ and $w(x) := (x_{p+1}, x_{p+2}, \ldots, x_n) \in W$. Let φ be admissible

[6]Different sorts of information condidtions are extensively covered in the literature. See, for example, (Roemer J, 1996) and (Sen A, 1974, p.13 -21) for discussion.

and, for $w \in W$, consider the restriction φ_w of φ to the p -dimensional plane $P_w \cong G_t$ through w orthogonal to W.

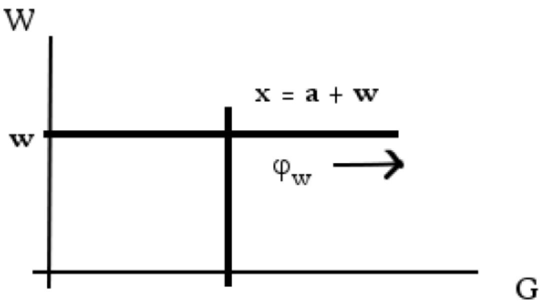

Figure 1:

Theorem 2. Suppose φ_w does not have a critical point for any w. Then the admissible welfare function φ is G_t acceptable if and only if it has the form:

$$\varphi(x) = \alpha(\langle C, h(x)\rangle + \lambda(w(x))) \tag{3}$$

for some monotonic $\alpha : IR \to IR$ and some $\lambda : W \to IR$ and some constant $C \in G_t$.

Proof of Theorem 2: See Appendix 1.

4. MULTIPLICATIONS

Consider the information condition that allows reports to be multiplied by a constant, not necessarily the same for each individual. In this case information on ratios is retained (Tsui and Weymark, 1997). The multiplication group $G_m \in IR^n_+$ acts non-trivially on the first p coordinates of $\tilde{X} \subset IR^n_+$ and trivially on the remainder with action

$$b \circ y = (b_1 y_1, b_2 y_2, \ldots, b_p y_p,\ y_{p+1}, \ldots, y_n)$$

for $y \in \tilde{X}$.

Corollary 1 of Theorem 2:[7] φ will be G_m -acceptable if and only if it has the form:

$$\varphi(y) = \tilde{\alpha}(\tilde{\lambda}(\tilde{w})\prod_{i=1}^{p} y_i^{C_i})$$

where $\tilde{w} = (y_{p+1}, \ldots, y_n) \in \tilde{W}$ for some $\tilde{\alpha} : IR \to IR$ and some $\tilde{\lambda} : \tilde{W} \to IR$ and some $C \in G_m$.

Proof: Make the change of coordinates $a_i \mapsto \ln b_i$ and $x_i \mapsto \ln y_i$. The idea, illustrated in Figure **2**, is to show that Theorem add now applies and then translate back to the original coordinates. Apply G_m to $\tilde{X}$ and then apply the change of coordinates to give $\ln by \mapsto \ln b + \ln y$. Using equation (main) we now have

$$\varphi = \alpha(\sum_{i=1}^{p} C_i \ln y_i + \lambda(\ln y_{p+1}, \ldots, \ln y_n))$$

and letting $\tilde{\lambda}(\tilde{w}) := e^{\lambda}$ and $\tilde{\alpha} = \alpha \circ \ln$ completes the proof.

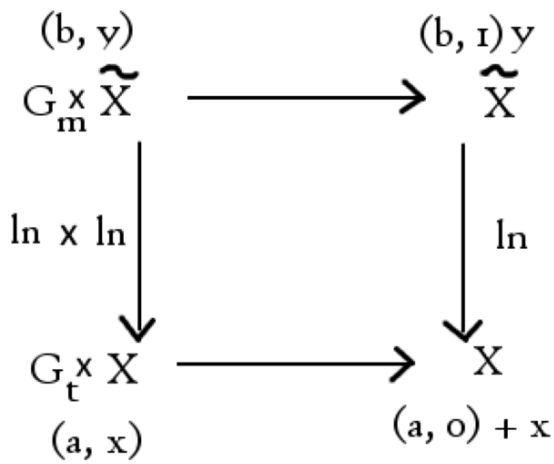

Figure 2:

5. EXPONENTS

An interesting question is whether there is an acceptable welfare function for non-linear transformations. Consider the condition that allows reports to be raised to a positive power, not necessarily the same for each individual. In this case information on ratios is retained after the change of coordinates $x_i \mapsto \ln z_i$. This transformation is given by a continuous group $G_p \in IR^n_+$ that acts on the first p coordinates of $\hat{X} \subset IR^n_+ + \mathbf{1}$ where $\mathbf{1}$ is the n -dimensional vector $(1,1,\ldots 1)^T$. This action is defined by

$$b \circ z = (z_1^{b_1}, z_2^{b_2}, \ldots, z_p^{b_p}, z_{p+1}, \ldots, z_n)$$

for $z \in \bar{X}$. Let $\bar{w} = (z_{p+1}, \ldots, z_n) \in \bar{W}$.

[7]This might be compared with Tsui and Weymarks result that under multiplications, or ratio scale transformations, the only acceptable welfare function is Cobb-Douglas (Tsui and Weymark, 1997).

Corollary 2 of Theorem 2: φ will be G_p -acceptable if and only if it has the form:

$$\varphi(z) = \bar{\alpha}(\bar{\lambda}(\bar{w})\prod_{i=1}^{p}(\ln z_i)^{C_i}$$

for some $\bar{\alpha} : IR_+ \to IR$ and some $\bar{\lambda} : \bar{W} \to IR$ and some $C \in G_p$.

Proof: Make the change of coordinates $a_i \mapsto \ln b_i$ and $x_i \mapsto \ln\ln z_i$ for all i. Following the previous proof $\ln\ln(z_i^{b_i}) = a_i + x_i$ and Theorem 2 can be applied since $\ln\ln z_i$ is well defined.

6. IMPOSSIBILITY

Consider any group of permissible transformations G_c with the property that its action contains two or more non-trivial actions from the set of transformations $\{G_t, G_m, G_p\}$. This gives:

Corollary 3 of Theorem 2: There is no G_c -acceptable φ.

Proof: This follows immediately from the fact that a G_c -acceptable φ would require at least two of the conditions set out on Theorem add, Corollary 1 and Corollary 2 to be satisfied simultaneously.

In this case we have an analogue to an Arrow type impossibility theorem. It will be noted that it is not necessary to allow any order preserving transformation to be permissible to get an impossibility result. From Corollary 1 and 2 we obtain the stronger result that impossibility holds under the weaker condition that permissible transformations are restricted to the affine transformation group.

The part of this result that deals with the affine transformation given by combining and G_m is familiar from the literature and has been discussed at length in a different context by, for example, (dAspremont and Gevers, 2002, Roemer J, 1996 and Sen A, 1974). It is well known from this literature that, unlike the translation and the multiplication the affine transformation does not permit interpersonal comparison (dAspremont and Gevers, 2002, p. 60). This could be extended to note that raising reports to a positive power also permits some interpersonal comparison. Combinations involving G_p do not permit such comparison.

6. CONCLUSION

This paper has treated choice as a mapping IR^n to IR and has developed the idea using continuity and a group invariance condition to identify the welfare functions that are acceptable under different information conditions. It should be stressed that there are no other acceptable functions under each information condition than those identified.

Among the questions that remain is what happens when preferences are interdependent as a result of an individual having a concern for the welfare of others, or being influenced by fashion or expectations. In this case the report of individual i would be a function of is preferences and the reports of some other set of individuals, say j and k. It would be possible to deal with this by examining transformation groups that do not act on each preference individually. Despite its difficulty, this might develop some interesting insights on choice under different forms of interdependency (beyond the scope of the present paper).

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APPENDIX 1

Proof of Theorem 2: The if part is immediate. For only if, we argue as follows. Fix $w \in W$ and let $K(w)$ be the kernel of the derivative at $x = w$ with respect to $x \in P_w$ of $\varphi_w : P_w \cong G_t \to IR$, written $d(\varphi_w)_w$. Because φ_w has no critical points, $K(w)$ is a $p-1$ dimensional vector subspace of G_t. Given $a \in G_t$, differentiating (accept) with respect to $x \in P_w$ at $x = w$ in the direction of any $\varpi \in K(w)$ gives

$$d(\varphi_w)_{a+w}(\varpi) = \psi_2(a, \varphi(w))d\varphi_w(\varpi) = 0$$

where ψ_2 denotes the derivative of ψ with respect to its second variable. Any $x \in P_w$ has the form $x = a + w$, and so it follows that $d(\varphi_w)_x$ vanishes on $K(w)$. So $\varphi_w(x)$ depends only on w and the projection of x orthogonal to $K(w)$. Because ϕ_w has no critical points $grad_x \ (\varphi_w)_w \neq \mathbf{0}$, and the unit vector C_w in the direction of this gradient is orthogonal to $K(w)$. So for any $x \in P_w$, $\varphi_w(x)$ depends only on w and $\langle C_w, x\rangle$.

As noted previously, any $x \in X$ has the form

$h(x)+w(x)$ where $h(x)\in G_t$ and $w(x)\in W$. So, for some $\chi_w : IR \to IR$,

$$\varphi(x)=\varphi_{w(x)}(x)=\chi_{w(x)}(\left\langle C_{w(x)},h(x)\right\rangle) \tag{4}$$

where χ_w is monotonic since φ_w has no critical points.

Using (4) to substitute for φ in equation (2) and writing $t := \varphi(x)$, we have

$$\chi_w(\left\langle C_w,a\right\rangle+\chi_w^{-1}(t))=\chi_w(\left\langle C_w,a\right\rangle+\left\langle C_w,h(x)\right\rangle)=\varphi(a+x)=\psi(a,t) \tag{5}$$

and it follows that, for given values of $a\in G$ and $t\in IR$, the left hand side is independent of w.

Let a be fixed and partially differentiate the left hand side of equation (expand) with respect to the coordinates w_i of w where $i=1,2,\ldots,n-p$. This gives

$$\frac{\partial\chi_w}{\partial w_i}_{\left\langle C_w,a\right\rangle+\chi_w^{-1}(t)}+\chi_w'(\left\langle C_w,a\right\rangle+\chi_w^{-1}(t)))(\left\langle\frac{\partial C_w}{\partial w_i},a\right\rangle+\frac{\partial\chi_w^{-1}(t)}{\partial w_i})\equiv 0.$$

Dividing through by $\chi_w'(\left\langle C_w,g\right\rangle+\chi_w^{-1}(t))$ and using the inverse function theorem we find that

$$\frac{1}{\chi_w'}\frac{\partial\chi_w}{\partial w_i}_{\left\langle C_w,a\right\rangle+\chi_w^{-1}(t)}+\left\langle\frac{\partial C_w}{\partial w_i},a\right\rangle\equiv-\frac{\partial\chi_w^{-1}(t)}{\partial w_i}=\frac{1}{\chi_w'}\frac{\partial\chi_w}{\partial w_i}_{\chi_w^{-1}(t)}$$

where the last equality follows from differentiating both sides of $u:=\chi_w^{-1}(t)\Leftrightarrow t=\chi_w(u)$ with respect to w_i.

It follows from the fact that C_w has unit length that $\left\langle\frac{\partial C_w}{\partial w_i},C_w\right\rangle\equiv 0$. Since the fixed value of a is arbitrarily chosen set $a=rC_w$ for $r\in IR$ an arbitrary constant. We then have

$$\frac{1}{\chi_w'}\frac{\partial\chi_w}{\partial w_i}_{r+\chi_w^{-1}(t)}=\frac{1}{\chi_w'}\frac{\partial\chi_w}{\partial w_i}_{\chi_w^{-1}(t)}$$

namely $\frac{1}{\chi_w'}\frac{\partial\chi_w}{\partial w_i}_{r+\chi_w^{-1}(t)}$ is independent of r. This means that for any fixed w we have

$$\frac{1}{\chi_w'}\frac{\partial\chi_w}{\partial w_i}=c_i(w) \tag{6}$$

for some constant $c_i(w)$ depending only on w.

Lemma. For some $\lambda : W\to IR$ we have $(c_1(w),c_2(w),\ldots,c_{n-p}(w))=grad\lambda(w)$.

Proof of Lemma. Rearrange equation (6) and differentiate with respect to w_j where $j\neq i$ and $j=1,2,\ldots,n-p$ to get

$$\frac{\partial^2\chi_w}{\partial w_i w_j}=\frac{\partial c_i(w)}{\partial w_j}\chi_w'+c_i(w)\frac{\partial\chi_w}{\partial w_j}=\frac{\partial c_i(w)}{\partial w_j}\chi_w'+c_i(w)c_j(w)\chi_w'$$

and hence $\frac{\partial c_i(w)}{\partial w_j}=\frac{\partial c_j(w)}{\partial w_i}$ which satisfies the equality of cross partials which is all we need.

It follows from the lemma and equation (constant) that $grad_W\chi_w(m)=\chi_w'(m)grad\lambda(w)$ for any $m\in IR$. If $u\mapsto(w(u),m(u))\in W\times IR$ is any smooth curve then

$$\frac{d}{du}(\chi_{w(u)}(m(u)))=\chi_{w(u)}'(m(u))\frac{dm}{du}+\left\langle grad_W\chi_w(m(u)),\frac{dw(u)}{du}\right\rangle \tag{7}$$

and from the expression for $grad_W\chi_w(m)$ this is

$$\chi_{w(u)}'((m(u))(\frac{dm}{du}+\left\langle grad\lambda(w(u)),\frac{dw(u)}{du}\right\rangle)$$

If $m(u)+\lambda(w(u))$ is constant then this expression is zero and from equation (derivativechi) we have $\chi_{w(u)}(m(u))$ constant. So we can set $\chi_w(m)=\alpha(m+\lambda(w))$ for some monotonic function $\alpha : IR\to IR$.

Now m is arbitrary. Set $m=\left\langle C_w,h\right\rangle$. By equation (varphi),

$$\varphi(x) = \chi_w(\langle C_w, h\rangle) = \alpha(\langle C_w, h\rangle + \lambda(w)).$$

So by equation (accept), for any $a \in G_t$,

$$\psi(a, \varphi(x)) = \varphi(a + x) = \alpha(\langle C_w, a\rangle + \langle C_w, h\rangle + \lambda(w)) = \alpha(\langle C_w, a\rangle + \alpha^{-1}(\varphi(x))) \quad (8)$$

namely $\psi(a, t) = \alpha(\langle C_w, a\rangle + \alpha^{-1}(t))$ where $t = \varphi(x)$ and the left hand side is independent of w for any given t. Since α is monotonic, $\langle C_w, a\rangle$ is independent of of w. This means that C_w is a constant $C \in G_t$, as required.

REFERENCES

Arrow, K. 1951. *Social Choice and Individual Values.* John Wiley. New York.

Arrow, K., and Sen, A., (eds). 2002. *Handbook of Social Choice and Welfare.* Vol. 1. Elsevier.

Coram, A., and Noakes, L. 2010. 'Social choice from an n dimensional euclidean space: a short impossibility proof.' *Australian Economic Papers.* December. 253-8 http://dx.doi.org/10.1111/j.1467-8454.2010.00400.x

d' Aspermont, C., and Gevers, L., 2002. 'Social welfare functions and interpersonal comparability,' in Arrow, K., and Sen, A., (eds). *Handbook of Social Choice and Welfare.* Vol. 1. Elsevier. http://dx.doi.org/10.1016/S1574-0110(02)80014-5

Le Breton, L. and Weymark, J. 2002 'Arrovian social choice on economic domains' in Arrrow, K., and Sen, A., (eds). 2002. *Handbook of Social Choice and Welfare.* Vol. 2. Elsevier.

Roemer, J. 1996. *Theories of Distributive Justice.* Harvard. Cambridge Mass.

Sen, A. 1974. 'Informational Bases of alternative welfare approaches,' in *Journal of Public Economics Economics.* 1947. 3 387-403.

Sen, A. 1986. 'Social choice theory,' in *Handbook of Mathematical Economics.* vol. 111. Elsevier. North Holland.

Tsui, K. and Weymark, J 1997., Social welfare orderings for ratio-scale measurable utilities,' *Economic Theory.* 10. 241-256. http://dx.doi.org/10.1007/s001990050156

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