Expectations, Taylor Rules and Liquidity Traps

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Abstract: This paper includes three equilibrium paths (Christiano *et al*. 2011; Werning 2012; Cochrane 2016) that model liquidity traps in a unified framework with expectations of duration of the zero lower bound and expectations of a Taylor-type rule outside of the trap, and finds that their appearance depends on these expectations. Other than that, Werning (2012)'s and Cochrane (2016)'s equilibrium paths require one more strong assumption and are thus arguably harder to observe in reality although Cochrane (2016)'s equilibrium path fits the recent data better.

Keywords: Liquidity traps, The zero lower bound, Equilibrium selection, Taylor rules.

1. INTRODUCTION

The liquidity trap is usually associated with output collapse, deflation, negative natural real interest rates and the zero lower bound on nominal interest rates binding in the literature, a scenario that can be generated from New-Keynesian models by selecting one of the many possible equilibrium paths that are used to explain the law of motion of an economy under some particular circumstances. Three equilibrium paths are observed in the literature. The path studied in Christiano *et al*. (2011) generates a bounded fall in both inflation and the output gap, the path studied in Werning (2012) an unbounded fall in both inflation and the output gap, and the path studied in Cochrane (2016) mild inflation and a small fall in output. Policy response suggestions are also found to be path-specific. Government spending, supply-side policies that promote market inefficiency, forward guidance that commits to near-zero interest rates for a certain period in the future are expansionary in the former two cases (Carlstrom *et al*. 2014; Eggertsson 2012, 2014; Woodford 2012) but contractionary in the latter one.

The recent Great Recession is followed by a long period of the zero lower bound on nominal interest rates, temporary deflation and then low inflation, a temporary output fall and then rise. Many papers, such as Weiland (2014), Negro *et al*. (2015) and Kiley (2016), together with empirical findings, point out the puzzlingly small effects of the suggested policy responses based on the former two cases, seemingly that the reality supports Cochrane (2016)'s equilibrium path that assumes that nominal interest rates follow a peg after exit from the zero lower bound and is built on the Cochrane (2011)'s critique that the Taylor-type interest rate feedback rule if used upon exit can only induce nominal explosions and nothing sensible can rule them out. This paper shows that nominal explosions can still arise even if assuming an interest rate peg. To be specific, the Taylor-type interest rate rule generates all but one stable solution, an interest rate peg some explosive and some stable solutions. This paper argues that the incapable of ruling out nominal explosions under Taylor rules cannot be justified as an argument to reject Taylor rules and to use interest rate pegs instead.

As long as such controversy exists, this paper believes that it can still include the Taylor-type interest rate feedback rule in New-Keynesian models and finds that all three equilibrium paths can be included in a unified framework and their appearance depends on expected duration of the zero lower bound and expected responsiveness of monetary policy to inflation upon exit when agents in a continuous-time version of the basic New-Keynesian model hear of the news that an adverse natural real interest rate shock buffets the model economy and the zero lower bound starts to bind for some periods of time. It finds that Christiano *et al*. (2011)'s unique equilibrium path prevails after ruling out all explosive solutions if expected duration of the zero lower bound is short or expected responsiveness to inflation upon exit is very active or both and if expected duration is long or expected responsiveness to inflation is less active upon exit or both, multiple stable equilibrium paths emerge after ruling out all explosive ones, among which Werning (2012)'s equilibrium path is selected if imposing a boundary condition and Cochrane (2016)'s equilibrium path prevails if equilibria are indexed and selected based on their fiscal consequences. The latter two equilibrium paths require one more strong assumption and are thus arguably harder to observe in reality even though Cochrane (2016)'s equilibrium path fits the recent data better.

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Note that Werning (2012) and Cochrane (2016) analyze liquidity traps in deterministic environment where the zero lower bound is assumed to bind for a known number of periods. The nominal interest rate follows a time- or state-varying peg, doesn't respond to inflation and is assumed to remain exogenous to the state of economy forever, which leads to dynamic indeterminacy, i.e. a continuum of bounded solutions. But the assumption of no endogenous response of monetary policy to inflation seems unrealistic. It has been well known that dynamic indeterminacy can also result from a passive response to inflation, a situation that may yield the same result, seems more realistic but cannot be depicted by a peg. This paper analyzes liquidity traps in a stochastic environment where there is always a fixed probability of exiting the zero lower bound each period, which gives an opportunity to depict monetary policies upon exit, such as including a Taylor-type interest rate rule. It can be shown in the following analysis that if policy response to inflation is expected to be less than one-for-one upon exit, indeterminacy always exists, Christiano *et al*. (2011)'s equilibrium path is excluded and Cochrane (2016)'s equilibrium path can be selected if considering fiscal consequences. In reality, the zero lower bound episode may occur again in the future. Although constrained, by making credible announcements about future stance towards inflation, monetary policy may lead to a small output gap and gentle inflation.

2. MODEL AND RESULT

The continuous-time version of the basic New-Keynesian model (Woodford 2003) can be described by the following two equations

$$\frac{dx_t}{dt} = \sigma[p\bar{r} + (1-p)\phi_\pi \pi_t - \pi_t]$$

$$\frac{d\pi_t}{dt} = \rho\pi_t - \kappa x_t$$

where defining $i_t - r_t = p\bar{r} + (1-p)\phi_\pi \pi_t$, it is assumed that as time moves forward by an infinitesimal step (dt), changes in the output gap (dx_t) are driven by inflation (π_t) and the mathematical expectation of the difference between nominal and natural real interest rates ($i_t - r_t$) with probability p of $r_t < 0$ that the zero lower bound on nominal interest rates binds ($i_t = -i$) and $i_t - r_t$ equals a positive number ($\bar{r}$) and with probability $1-p$ of $r_t = 0$ that $i_t - r_t = i_t$ follows a Taylor-type rule ($\phi_\pi \pi_t$). p stands for not only expected persistence of the r_t-drop and thus expected severity of the shock depicted by such a drop given $\bar{r}$ but also expected duration of the zero lower bound, the larger the value of p the longer the expected duration. Also assumed is that the model economy starts in a liquidity trap and exits from it at some date T.

To solve for equilibrium inflation and the output gap, differentiate the Phillips curve and substitute for $\frac{dx_t}{dt}$ from the IS curve to obtain

$$\frac{d^2\pi_t}{dt^2} - \rho\frac{d\pi_t}{dt} + \kappa\sigma[(1-p)\phi_\pi - 1]\pi_t = -\kappa\sigma p\bar{r}$$

which can be rewritten in an operator form of

$$\left(\frac{d}{dt} - \lambda_1\right)\left(\frac{d}{dt} - \lambda_2\right)\pi_t = -\kappa\sigma p\bar{r}$$

When $0 < p < 1 - \frac{1}{\phi_\pi}$, it can be verified that $0 < \lambda_i = \frac{\rho \pm \sqrt{\rho^2 - 4\kappa\sigma[(1-p)\phi_\pi - 1]}}{2} < 1$ for $i = 1,2$, and equilibrium inflation and the output gap are

$$\begin{aligned}\pi_t &= C_1 e^{\lambda_1 t} - C_2 e^{\lambda_2 t} - \frac{\kappa\sigma p\bar{r}}{\lambda_1 - \lambda_2}\int_{s=t}^{T}[e^{-\lambda_2(s-t)} - e^{-\lambda_1(s-t)}]\,ds \\ &= -\frac{\kappa\sigma p\bar{r}}{\lambda_1 - \lambda_2}\left\{\frac{1}{\lambda_2}[1 - e^{-\lambda_2(T-t)}] - \frac{1}{\lambda_1}[1 - e^{-\lambda_1(T-t)}]\right\} < 0\end{aligned}$$

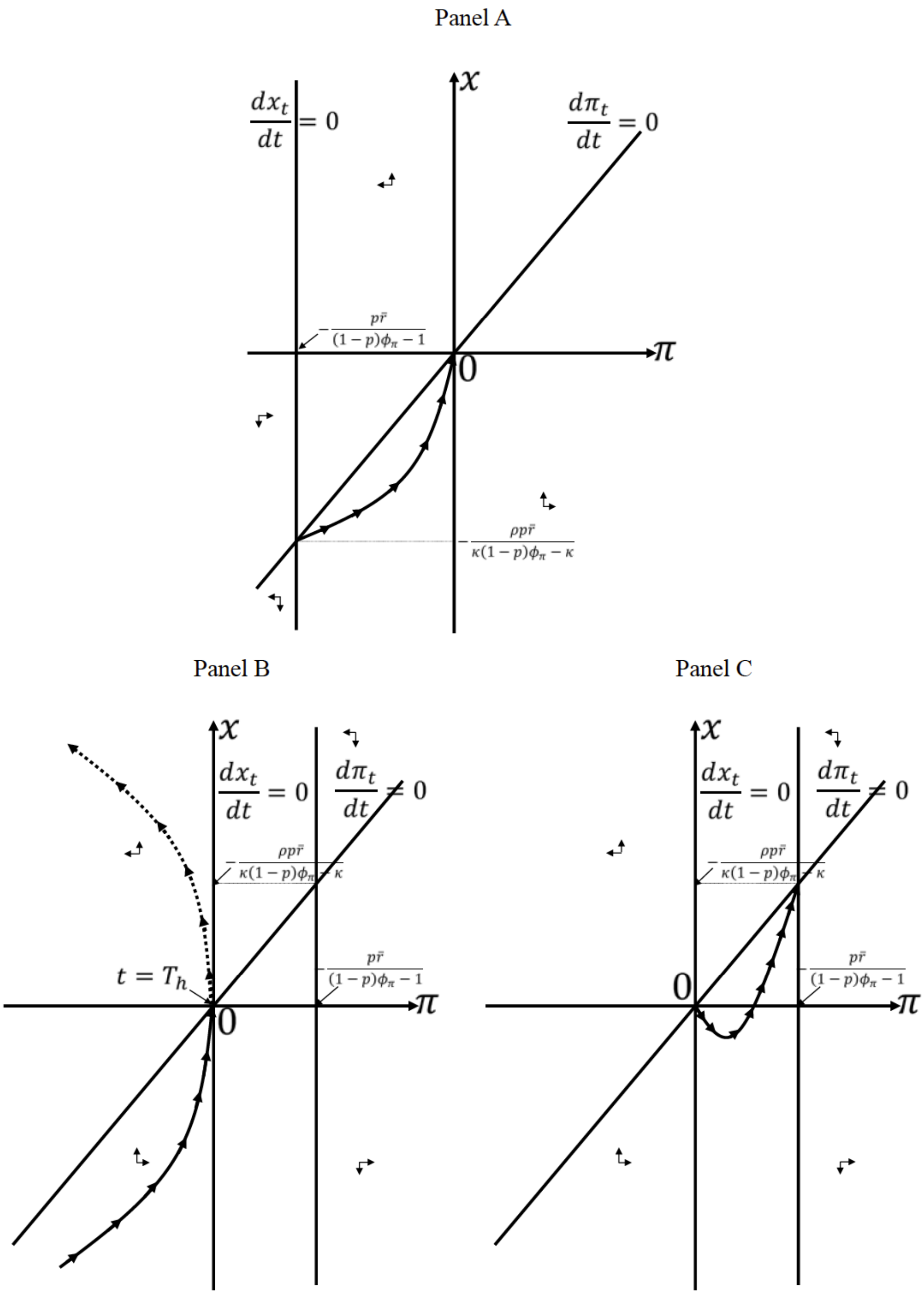
$$\begin{aligned}\kappa x_t &= \lambda_2 C_1 e^{\lambda_1 t} - \lambda_1 C_2 e^{\lambda_2 t} - \frac{\kappa\sigma p\bar{r}}{\lambda_1 - \lambda_2}\int_{s=t}^{T}[\lambda_1 e^{-\lambda_2(s-t)} - \lambda_2 e^{-\lambda_1(s-t)}]ds \\ &= -\frac{\kappa\sigma p\bar{r}}{\lambda_1 - \lambda_2}\left\{\frac{\lambda_1}{\lambda_2}[1 - e^{-\lambda_2(T-t)}] - \frac{\lambda_2}{\lambda_1}[1 - e^{-\lambda_1(T-t)}]\right\} < 0\end{aligned}$$

where $C_1 e^{\lambda_1 t} - C_2 e^{\lambda_2 t} = \lambda_2 C_1 e^{\lambda_1 t} - \lambda_1 C_2 e^{\lambda_2 t} = 0$, which is that all explosive solutions are ruled out with $C_1 = C_2 = 0$. It can be seen that both inflation and the output gap drop in the face of a r_t-drop. More importantly, the solution is unique, even with the zero lower bound binding, in that a quick exit from the liquidity trap is expected because of low p values and a Taylor-type interest rate feedback rule is believed to stabilize the economy soon. For $t \ge T$, $(\pi_t, x_t) = (0,0)$. For $t < T$, (π_t, x_t) decreases in *T*, and as $T \to \infty$, $(\pi_t, x_t) = \left(-\frac{p\bar{r}}{(1-p)\phi_\pi - 1}, -\frac{\rho p\bar{r}}{\kappa(1-p)\phi_\pi - \kappa}\right)$. This equilibrium path is depicted in Panel A and is also studied in Christiano *et al*. (2011) in a discrete-time model. An increase in p shifts $\frac{dx_t}{dt} = 0$ curve leftward and harms the economy.

When $p = 1 - \frac{1}{\phi_\pi}$, model collapses and no solution exists. It is also found that the more active the monetary policy response to inflation after exit from the zero lower bound, the larger the value of p needed to reach this singularity and the more likely the Christiano *et al*. (2011)'s equilibrium path holds.

When $1 - \frac{1}{\phi_\pi} < p \le 1, \kappa\sigma[(1-p)\phi_\pi - 1]$ turns negative, and the operator form now becomes

$$\left(\frac{d}{dt} - \lambda^f\right)\left(\frac{d}{dt} + \lambda^b\right)\pi_t = -\kappa\sigma p\bar{r}$$



where $\lambda^f = \dfrac{\rho + \sqrt{\rho^2 - 4\kappa\sigma[(1-p)\phi_\pi - 1]}}{2} > 0$ and $\lambda^b = \dfrac{\sqrt{\rho^2 - 4\kappa\sigma[(1-p)\phi_\pi - 1]} - \rho}{2} > 0.$ Equilibrium solutions are

$$\pi_t = C_1 e^{-\lambda^b t} - C_2 e^{\lambda^f t} + \frac{\kappa\sigma p\bar{r}}{\lambda^f + \lambda^b}\left[\int_{s=T_l}^{t} e^{-\lambda^b(t-s)}ds + \int_{s=t}^{T_h} e^{-\lambda^f(s-t)}ds\right]$$
$$= C_1 e^{-\lambda^b t} + \frac{\kappa\sigma p\bar{r}}{\lambda^f + \lambda^b}\left\{\frac{1}{\lambda^b}[1 - e^{-\lambda^b(t-T_l)}] + \frac{1}{\lambda^f}[1 - e^{-\lambda^f(T_h-t)}]\right\}$$
$$\kappa x_t = \lambda^f C_1 e^{-\lambda^b t} - \lambda^b C_2 e^{\lambda^f t} + \frac{\kappa\sigma p\bar{r}}{\lambda^f + \lambda^b}$$
$$\left[\lambda^f \int_{s=T_l}^{t} e^{-\lambda^b(t-s)}ds - \lambda^b \int_{s=t}^{T_h} e^{-\lambda^f(s-t)}ds\right]$$
$$= \lambda^f C_1 e^{-\lambda^b t} + \frac{\kappa\sigma p\bar{r}}{\lambda^f + \lambda^b}\left\{\frac{\lambda^f}{\lambda^b}[1 - e^{-\lambda^b(t-T_l)}] - \frac{\lambda^b}{\lambda^f}[1 - e^{-\lambda^f(T_h-t)}]\right\}$$

where $T_h - T_l = T$ and $C_2 e^{\lambda^f t} = \lambda^b C_2 e^{\lambda^f t} = 0$ with $C_2 = 0$ to rule out all explosive solutions. C_1 can be any number, implying that the dynamic system is indeterminate. If $\phi_\pi \le 1,$ indeterminacy always holds.

One equilibrium path that is studied in Werning (2012) is constructed by assuming a boundary condition $(\pi_t, x_t) = (0,0)$ for $t \ge T$ that requires C_1 to be

$$0 = C_1 e^{-\lambda^b T_h} + \frac{\kappa\sigma p\bar{r}}{\lambda^f + \lambda^b}\frac{1}{\lambda^b}\left[1 - e^{-\lambda^b T}\right]$$

and equilibrium inflation and the output gap are

$$\pi_t = -\frac{\kappa\sigma p\bar{r}}{\lambda^f + \lambda^b}\frac{1}{\lambda^b}\left[e^{\lambda^b(T_h-t)} - e^{-\lambda^b(t-T_l)}\right]$$
$$+ \frac{\kappa\sigma p\bar{r}}{\lambda^f + \lambda^b}\left\{\frac{1}{\lambda^b}[1 - e^{-\lambda^b(t-T_l)}] + \frac{1}{\lambda^f}[1 - e^{-\lambda^f(T_h-t)}]\right\}$$
$$\kappa x_t = -\frac{\kappa\sigma p\bar{r}}{\lambda^f + \lambda^b}\frac{\lambda^f}{\lambda^b}\left[e^{\lambda^b(T_h-t)} - e^{-\lambda^b(t-T_l)}\right]$$
$$+ \frac{\kappa\sigma p\bar{r}}{\lambda^f + \lambda^b}\left\{\frac{\lambda^f}{\lambda^b}[1 - e^{-\lambda^b(t-T_l)}] - \frac{\lambda^b}{\lambda^f}[1 - e^{-\lambda^f(T_h-t)}]\right\}$$

which implies that initial inflation and the output gap are unbounded as $T \to \infty$. This equilibrium path is depicted in Panel B. Changes in p can shift $\frac{dx_t}{dt} = 0$ curve but severity of the liquidity trap is mainly determined by actual (*T*) rather than expected (p) duration of the zero lower bound.

Another equilibrium path that is studied in Cochrane (2016) is constructed by setting initial inflation to zero $\pi_{T_l} = 0$ That requires C_1 to be

$$0 = C_1 e^{-\lambda^b T_l} + \frac{\kappa\sigma p\bar{r}}{\lambda^f + \lambda^b}\frac{1}{\lambda^f}\left[1 - e^{-\lambda^f T}\right]$$

and equilibrium inflation and the output gap are

$$\pi_t = -\frac{\kappa\sigma p\bar{r}}{\lambda^f + \lambda^b}\frac{1}{\lambda^f}\left\{e^{-\lambda^b(t-T_l)} - e^{[-\lambda^b(t-T_l) - \lambda^f T]}\right\}$$
$$+ \frac{\kappa\sigma p\bar{r}}{\lambda^f + \lambda^b}\left\{\frac{1}{\lambda^b}[1 - e^{-\lambda^b(t-T_l)}] + \frac{1}{\lambda^f}[1 - e^{-\lambda^f(T_h-t)}]\right\}$$
$$\kappa x_t = -\frac{\kappa\sigma p\bar{r}}{\lambda^f + \lambda^b}\left\{e^{-\lambda^b(t-T_l)} - e^{[-\lambda^b(t-T_l) - \lambda^f T]}\right\}$$
$$+ \frac{\kappa\sigma p\bar{r}}{\lambda^f + \lambda^b}\left\{\frac{\lambda^f}{\lambda^b}[1 - e^{-\lambda^b(t-T_l)}] - \frac{\lambda^b}{\lambda^f}[1 - e^{-\lambda^f(T_h-t)}]\right\}$$

which implies an initial bounded drop of the output gap and it can be verified that the drop is exacerbated with large values of p. The output gap then rises and a boom is created even in a liquidity trap. Inflation remains positive throughout the trap and decreases in p. With the zero lower bound binding, real interest rates are negative, contrary to the liquidity trap scenario defined by Christiano *et al*. (2011), Woodford (2011), Eggertsson (2011), Werning (2012) and many others as "a situation where negative real interest rates are needed to obtain the first best allocation."

3. CONCLUSION

This paper argues that emergence of explosive equilibrium solutions cannot be used as an argument to reject the Taylor-type interest rate feedback rule and to use an interest rate peg instead, since the latter also gives rise to explosive solutions. It includes the rule in a continuous-time version of the basic New-Keynesian model with an adverse natural real interest rate shock that puts the model economy into a liquidity trap, and finds that the three equilibrium paths (Christiano *et al*. 2011; Werning 2012; Cochrane 2016) that are observed in the literature can be included in a unified framework and their appearance depends on expected duration of the zero lower bound and expected responsiveness of monetary policy response to inflation. Other than that, Werning (2012)'s and Cochrane (2016)'s equilibrium paths require one more strong assumption and are thus arguably harder to observe in reality though Cochrane (2016)'s equilibrium path seems to fit the recent data better. In reality, the zero lower bound episode may occur again in the future. Suppose fiscal policy considerations are properly accounted for. This paper shows that if the public is convinced to expect a passive response to inflation upon exiting the zero lower bound, although constrained, monetary policy can lead to a small output gap and gentle inflation as depicted in Cochrane (2016).

REFERENCES

Carlstrom, Charles T., Timothy S. Fuerst, and Matthias Paustian. 2014. "Fiscal Multipliers under an Interest Rate Peg of Deterministic versus Stochastic Duration." Journal of Money, Credit and Banking vol. 46: 1293-1312. https://doi.org/10.1111/jmcb.12141

Christiano, Lawrence, Martin Eichenbaum, and Sergio Rebelo. 2011. "When Is the Government Spending Multiplier Large?" Journal of Political Economy vol. 119(1): 78-121. https://doi.org/10.1086/659312

Cochrane, John H. 2011. "Determinacy and Identification with Taylor Rules." Journal of Political Economy vol. 119 (3): 565-615. https://doi.org/10.1086/660817

Cochrane, John H. 2016. "The New-Keynesian Liquidity Trap." URL http://faculty.chicagobooth. edu/john.cochrane/research/papers/zero_bound_elsarticle-harv.pdf. Manuscript.

Eggertsson, Gauti B. 2011. "What Fiscal Policy is Effective at Zero Interest Rates?" NBER Macroeconomics Annual vol. 25: 59-112. https://doi.org/10.1086/657529

Eggertsson, Gauti B. 2012. "Was the New Deal Contractionary?" American Economic Review, American Economic Association vol. 102(1): 524-55. https://doi.org/10.1257/aer.102.1.524

Eggertsson G., A. Ferrero, and A. Raffo. 2014. "Can structural reforms help Europe?" Journal of Monetary Economics vol. 61(C): 2-22. https://doi.org/10.1016/j.jmoneco.2013.11.006

Kiley, Michael T. 2016. "Policy Paradoxes in the New Keynesian Model." Review of Economic Dynamics vol. 21: 1-16. https://doi.org/10.1016/j.red.2016.03.002

Negro, Marco Del, Marc Giannoni, and Christina Patterson. 2015. "The Forward Guidance Puzzle." Manuscript.

Wieland, Johannes. 2014. "Are Negative Supply Shocks Expansionary at the Zero Lower Bound? " Manuscript.

Werning, I. 2012. "Managing a liquidity trap: Monetary and fiscal policy." Manuscript.

Woodford, Michael. 2003. Interest and prices. Princeton N. J.: Princeton University Press.

Woodford, Michael. 2011. "Simple Analytics of the Government Expenditure Multiplier." American Economic Journal: Macroeconomics vol. 3(1): 1-35. https://doi.org/10.1257/mac.3.1.1

Woodford, Michael. 2012. "Methods of Policy Accommodation at the Interest-Rate Lower Bound." Proceedings - Economic Policy Symposium - Jackson Hole, Federal Reserve Bank of Kansas City 185-288.

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