Inflation and Cash

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Abstract: Cash demand is investigated by means of mathematical analysis. The optimal cash amount a person needs in order to minimize the effect of inflation and maximize percent money is given here. The corresponding number of cash transfers for a period is calculated. The corresponding formulas are presented and proved.

Keywords: Baumol-Tobin formula, cash withdrawal, average cash, cash demand, inflation rate.

I. INTRODUCTION

The inflation was not considered earlier because it was not significant in developed countries not long ago, and so insignificant for cash formation. Now there is high inflation (even hyperinflation) in many countries. So we tried to investigate cash demand taking inflation into account. In this case the analysis is not so easy.

It is well known that price rise can be calculated by formulae

$$P_n = P_0\left(1+h_1\right)\left(1+h_2\right)...\left(1+h_n\right).$$

$\left(1+h_k\right)$ is price index; P_0, P_1 are prices initial and final price levels correspondingly. The average inflation rate can be found from equation $P_n = P_0\left(1+h\right)^n$.

$$h = \sqrt[n]{\Pi_{k=1}^{n}\left(1+h_k\right)} - 1.$$

If customer makes purchases m times in a period he would lose by inflation less money. In this case rate of inflation H can be calculated as follows.

$$h = \left(1+\frac{H}{m}\right)^m - 1; \; H = m\left(\sqrt[m]{1+h} - 1\right).$$

In case of insignificant inflation customer takes off the same sum of money every time. If inflation is large enough a person must take inflation into consideration and withdraw more money every other time.

Any customer has a choice. He can withdraw money m times in equal parts and he can increase withdrawing every other time in order to compensate inflation. Quantity of money in this case can be calculated as follows.

$$Y_1 = \frac{P_0}{m} + \frac{P_0}{m}\left(1+\frac{H}{m}\right) + \frac{P_0}{m}\left(1+\frac{H}{m}\right)^2 + ... + \frac{P_0}{m}\left(1+\frac{H}{m}\right)^{m-1};$$

$$Y_1 = \frac{P_0\left(\left(1+\frac{H}{m}\right)^m - 1\right)}{H} = P_0\frac{h}{H}.$$

Let us consider simple case. Bank is visited two times, m = 2. At the beginning of the year customer withdraws $y_1 = \frac{P_0}{2}$, in half of a year withdrawing amount is equal to $y_2 = \frac{P_0}{2}\left(1+\frac{H}{2}\right)$. Average sum of money during the first half of a year is equal to $\frac{P_0}{4}$ and in the second half it is equal to $\frac{P_0}{4}\left(1+\frac{H}{2}\right)$. So annual average sum would be equal to $\left(\frac{y_1}{2}+\frac{y_2}{2}\right):2 = \frac{P_0}{8}\left(1+\left(1+\frac{H}{2}\right)\right)$. If customer visits bank more frequently the average sum would be less but the share of lost percent would be reduced too.

If money is taken off in equal parts at the beginning of a year and in a half of a year than an average sum would be equal to $\frac{Y_2}{4} = \frac{P_0\left(1+i\right)}{4}$.

If bank is visited m times a year with taking inflation into account an average sum would be equal to

$$\frac{1}{2}\cdot\frac{\left(y_1+y_2+...+y_m\right)}{m} = \frac{P_0}{m}\left(1+\left(1+\frac{H}{m}\right)++...+\left(1+\frac{H}{m}\right)^{m-1}\right) = \frac{P_0}{2m}\cdot\frac{\left(\left(1+\frac{H}{m}\right)^m - 1\right)}{H}.$$

In case of withdrawing money in equal parts an average sum is equal to $\frac{Y_2}{2m} = \frac{P_o}{2m}\left(1+i\right)$.

Sometimes in some places there appear problems with cash (lack of change at the shops, shortage of money in ATM and so on). So the investigation of cash

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demand is not an idle question. If we do not take inflation into consideration, than optimal value of average cash was calculated by Baumol and Tobin (Baumol, 1952; Tobin, 1956). The cash formation was discussed also in Gregory Mankiw's classic monograph (Mankiw, 1994). The empiric investigations of cash demand were made by M. Goldfield, 1973 and D. Laidlier, 1985. All abovementioned studies were founded on proposition of absence of inflation.

The inflation was not before considered because it was not significant in developed countries not long ago, and so insignificant for cash formation. Now there is high inflation (even hyperinflation) in many countries. So we tried to investigate cash demand taking inflation into account. In this case the analysis is not so easy. We used differential calculus and series to obtain and there is high inflation (even prove formulas for calculating cash flow. It is closely related to the question of realistic calculation of inflation itself. In case of higher inflation it is necessary to take into account the continuous character of price rises (Popov and Semenov, 2009). Now at the time of crisis it is impossible to ignore inflation. It is unreasonable to take off the same amount of money every time. In order to compensate rise of prices one must take off more now than previously. Our main task is to investigate optimal cash withdrawals and average cash under this condition.

II. MONEY WITHDROWAL IN EQUAL PARTS

If we do not take inflation into consideration, then the problem of calculating necessary cash will be analogous to well known problem of calculating storage cost. Let us assume that some person comes to the bank m times in a year and takes off the same sum of money $\frac{Y_2}{m}$ in cash. Here Y_2 is his sum of money on deposit.

As it is shown on Figure **1**, the cash in the purse of a person changes from $\frac{Y_2}{m}$ to zero and average value of person's cash is equals to $\frac{Y_2}{2m}$.

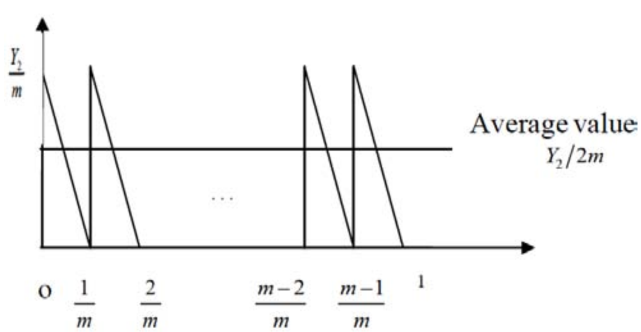

Figure 1: Person's cash as he comes to the bank m times and takes off the same amount of money.

Let i be a rate of interest. Then taking off money gives the lack of interest which equals to $\frac{iY_2}{2m}$. Side expenses (transport etc) on visiting bank and taking off money are equal to F. So the overall loss J_2, including lack of interest side expenses can be calculated by the following formula.

$$J_2 = \frac{iY_2}{2m} + Fm$$

Calculating the derivative $\frac{dJ_2}{dm} = -\frac{iY_2}{2m^2} + F$ and solving equation $-\frac{iY_2}{2m^2} + F = 0$ we have the following formulae for calculating m_0 and optimal value of average cash $\frac{iY_2}{2m}$.

$$m_0 = \sqrt{\frac{iY_2}{2F}}, \quad \frac{Y}{2m_0} = \sqrt{\frac{Y_2 F}{2i}}.$$

These are the main formulae of Baumol-Tobin transaction demand for cash model. This model deals with cash store formation.

At the time of crisis in is impossible to ignore inflation. It is unreasonable to take off the same amount of money every time. In order to compensate rise of prices one must take off more now than previously. Our main task is to investigate optimal cash takes off and average cash under these conditions.

III. MONEY WITHDROWAL WITH TAKING INFLATION INTO ACCOUNT

First of all consider rate of inflation h and real interest rate $r = \frac{i-h}{i+h}$. Under condition of full indexation, i.e. $i = h$ the formulae of Baumol-Tobin will be as follows $m_0 = \sqrt{\frac{hY_2}{2F}}$, $\frac{Y}{2m_0} = \sqrt{\frac{Y_2 F}{2h}}$. These formulae define optimal cash demand under inflation.

Let us assume for example that $Y_2 = 480$ (USD), $F = 0,2$. Than if $h = 0,1(10\%)$ than optimal number of take off money $m_0 \approx 11$ and optimal cash demand $\frac{Y}{2m_0} \approx 21,91$ (USD). If $h = 0.5(50\%)$ than $m_0 \approx 24$, $\frac{Y}{2m_0} \approx 9.8$ (USD).

Now we will follow more pragmatic approach. The person takes off more money every time in order to compensate inflation. Let H be nominal inflation rate and N_0 be initial price level (price of consumer basket). It is easy to calculate the necessary amount of money Y_1 on deposit.

$$Y_1 = \frac{P_0}{m} + \frac{P_0}{m}\left(1+\frac{H}{m}\right) + \frac{P_0}{m}\left(1+\frac{H}{m}\right)^2 + \ldots + \frac{P_0}{m}\left(1+\frac{H}{m}\right)^{m-1};$$

$$Y_1 = \frac{P_0\left(\left(1+\frac{H}{m}\right)^m - 1\right)}{H}.$$

Let h be effective inflation rate. $h = \left(1+\frac{H}{m}\right)^m - 1$. It follows $Y_1 = N_0 \cdot \frac{h}{H}$.

Average amount of money in the purse equals to $S = \frac{Y_1}{2m}$. Graf of this function is shown on Figure **2**.

Lack of interest under condition of equality of rate of inflation and interest rate equals to

$$S - \frac{S}{\left(1+\frac{H}{h}\right)^m} = \frac{Y}{2m}\left(1 - \frac{1}{\left(1+\frac{H}{h}\right)^m}\right)$$

If we add to this value side expenses on visiting bank we will find the overall loss J_1 of taking off money m times

$$J_1 = \frac{Y_1}{2m}\left(1 - \frac{1}{\left(1+\frac{H}{h}\right)^m}\right) + Fm.$$

Derivative of a function $J_1 = J_1(m)$ equals to $\frac{dJ_1}{dm} = \frac{Y_1}{2}\left(\frac{-\left(1+\frac{H}{h}\right)^{m+1} + \left(1+\frac{H}{h}\right) + m\left(1+\frac{H}{h}\right)\ln\left(1+\frac{H}{h}\right) - H}{}\right) + F$. As a rule $\frac{H}{m} \ll 1$. (It is untrue only in case of hyperinflation. in this case economy is in collapse). So we have the next approximations

$$\left(1+\frac{H}{m}\right)^{m+1} \approx \left(1+\frac{H}{m}\right)^m, \quad \left(1+\frac{H}{m}\right) \approx 1, \quad m\left(1+\frac{H}{m}\right)\ln\left(1+\frac{H}{m}\right) \approx 0,$$

$$(1-H) \approx 1.$$

Let us substitute these approximations to the expression of the derivative $\frac{dJ_1}{dm}$. We'll obtain the approximation of derivative $\frac{dJ_1}{dm} \approx \frac{Y_1}{2}\left(\frac{-\left(1+\frac{H}{m}\right)^m + 1}{m^2\left(1+\frac{H}{m}\right)^m}\right) + F$.

We'll find m^2 by solving the equation $\frac{dJ_1}{dm} = 0$.

$$m^2 = \frac{Y_1}{2}\left(\frac{-\left(1+\frac{H}{m}\right)^m + 1}{m^2\left(1+\frac{H}{m}\right)^m}\right).$$

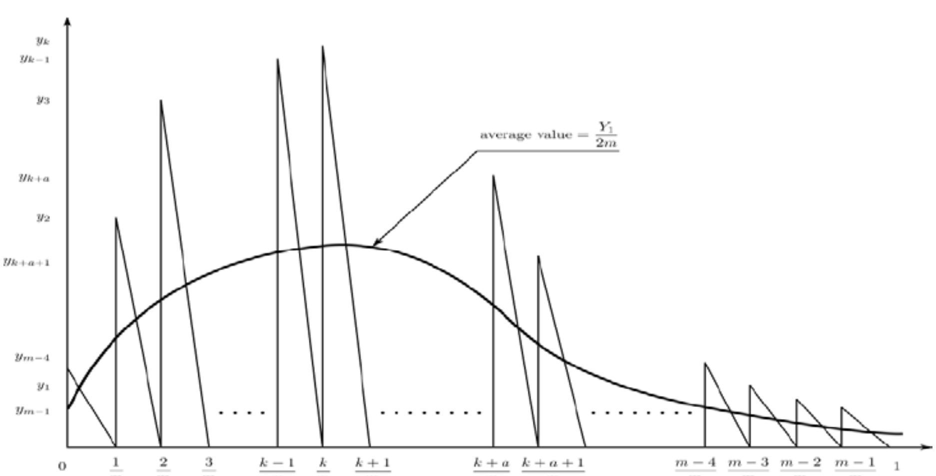

Figure 2: Person's cash as he comes to the bank m times and takes off unequal amount of money.

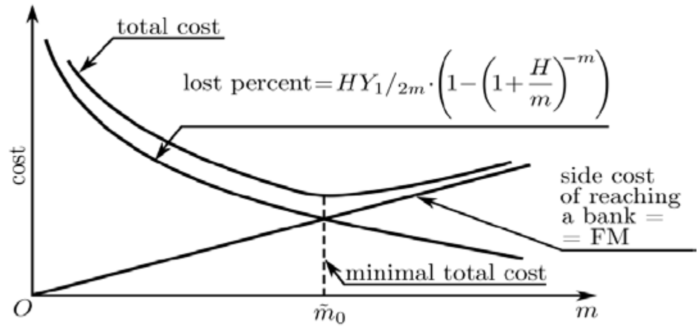

Figure 3: Cash storage cost.

As we have $\left(1+\frac{H}{m}\right)^m \approx e^H$ for large m so $m^2 \approx \frac{Y_1}{2}\cdot\frac{e^H-1}{e^H}$.

$e^H \approx 1+H$ for $H \ll 1$. $m^2 \approx \frac{Y_1}{2F}\cdot\frac{H}{1+H} \approx \frac{Y_1\cdot H}{2F}$. So the optimal number of taking off money m_0 and optimal cash $\frac{Y_1}{2m_0}$ are as follows approximately.

$$\widetilde{m_0} = \sqrt{\frac{Y_1 H}{2F}},\ \frac{Y_1}{2\widetilde{m_0}} = \sqrt{\frac{Y_1\cdot F}{2H}}.$$

It easy to see that minimum of function J_1 is $\widetilde{m_0}$.

First term (lost interest) $f_1(m) = \frac{Y_1}{2m}\left(1-\frac{1}{\left(1+\frac{H}{m}\right)^m}\right)$ is decreasing function. Second term (side expenses) $f_2(m) = Fm$ is increasing linear function. It follows that function $J_1 = f_1 + f_2$ has one point of extremum which is minimum. See Figure **3**.

IV. COCLUSIONS

1. If $Y_1 = Y_2 = Y$ than $\frac{Y}{2\widetilde{m_0}} - \frac{Y}{2m_0} = \sqrt{\frac{YF}{2}}\left(\frac{1}{\sqrt{H}}-\frac{1}{\sqrt{h}}\right) > 0$ i.e. a person who takes inflation into consideration has more cash than a person who ignore it.

If government index wages at the beginning of the year according to expected inflation than overall cash would be less in comparison with situation than government index wages several times in a year. The difference is insignificant if inflation is low, as in this case $H \approx h$, and will increase with growing inflation.

2. Increase of cost of consumer basket for person takes inflation into consideration is defined by formula

$$Y_1 = P_0\frac{h}{H} = P_0\left(1+\left(\frac{h}{H}-1\right)\right) = P_0(1+X).$$

Let us call $X = \frac{h}{H}-1$ inflation rate of constantly filled consumer basket.

For person who pays at the end of the period the cost of consumer basket is given by formula $Y_2 = N_0(1+h)$. The rate of inflation h is defined by this formula $h = \frac{Y_2 - N_0}{N_0}$ and h is equal to index of consumer basket prices minus one.

Now we will show that $h > X$. Actually.

$$h > \frac{h}{H}-1 \Leftrightarrow \frac{h}{H} < h+1 \Leftrightarrow h < (h+1)H \Leftrightarrow$$
$$h < (h+1)m\left(\sqrt[m]{1+h}-1\right) < (h+1)m\left(1+\frac{h}{m}-1\right)$$
$$\Leftrightarrow h < (h+1)h \Leftrightarrow 1 < h+1.$$

Person who visit shop m in a year and buy goods and services for current prices feels less inflation than officially given inflation. The concept of constantly filled basket is more realistic, because any person prefers to pay regularly. Official rate of inflation needs correction. It is suitable to use $X = \frac{h}{H}-1$ as a rate of inflation instead of h (Popov and Semenov, 2009).

3. The overall cash decrease as inflation grow according to the low $\frac{Y}{2\widetilde{m_0}} = k\cdot\frac{1}{\sqrt{H}}$ where $k = \sqrt{\frac{Y_1 F}{2}}$.

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