Successive Oligopolies in a Pure Exchange Economy

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Abstract: In this paper we assume a market structure in which there are whole-sellers, retailers and consumers. The product sold to the consumers is initially endowed with the whole-sellers. The whole-sellers value both the product and money. The retailers are not endowed with anything at all. The retailers submit bids in units of money to the whole-sellers from the revenue earned by selling the product to the consumers. The whole-sellers offer the product to the retailers who in turn sell it to the consumers. The retailers care only for money.

In this model there is a trivial equilibrium with no bids or offers being submitted. In addition we establish that a unique non-trivial equilibrium exists in which every trader participates in the market. Further, such an equilibrium is symmetric- all whole-sellers offer the same quantity of the product and all retailers bid the same quantity of money. We also obtain some comparative static results.

Keywords: Successive oligopolies, bilateral oligopoly, non-trivial equilibrium, comparative statics.

INTRODUCTION

The study of imperfect competition has usually taken place in the context of profit maximizing producers of a manufactured output who sell their output in a market where buyers are price takers. The producers are assumed to incur costs of production that is determined by a cost function. The market for inputs is assumed to be perfectly competitive with the input demand curves of the producers being perfectly elastic.

There are many ways in which the assumption of perfect competition in the input market can be relaxed. One of them is to assume that the input demand functions are downward sloping and the sellers of the inputs are oligopolists in the input market. In such a model there are upstream firms who manufacture inputs and sell then to the downstream firms. The downstream firms use these inputs to produce output which is sold to consumers. The costs incurred by the downstream firms are the revenue that accrues to the upstream firms. Each seller is an oligopolist in the market in which he sells his product.

Another variation which also implies relaxing the assumption of perfect competition is available in Gabszewicz, Laussel, van Yepersele and Zanaj (2007) and more recently Gabszewicz and Zanaj (2011). Here the interaction between the upstream and downstream firms is modeled as a bilateral oligopoly. A model of bilateral oligopoly as a strategic market game in the pure trade context based on the seminal works of Lloyd Shapley and Martin Shubik (Shapley (1976); Shapley and Shubik (1977); Shubik (1973)) is available in Gabszewicz and Michel (1997). In Gabszewicz *et al*. (2007), each downstream firm submits a bid to the upstream firms and each upstream firm offers inputs to the downstream firms. With inputs from the upstream firms, the downstream firms produce outputs which they sell to the consumers in an oligopolistic market. Part of the revenue that the downstream firms receive from the consumers is used for the bids that they submit to the upstream firms. The rest is their profit. In Gabszewicz *et al*. (2007) it is assumed that both the upstream as well as the downstream firms are profit maximizers and both, the input as well as the output, are manufactured from other inputs. While the downstream firms buy their inputs from the upstream firms with a portion of the revenue they earn from selling the output to the consumers, the upstream firms recover their costs of manufacture from the bids that the downstream firms submit.

In this paper we deviate from the assumption that the product sold to the consumers is manufactured by the downstream firms. Instead, we assume that the product sold to the consumers is initially endowed with the upstream firms. We call this product Y. The upstream firms value both Y and another commodity X, the latter being the numeraire commodity or money. The downstream firms are not endowed with anything at all. The downstream firms act as intermediaries between the upstream firms and the consumers. Hence the upstream firms are akin to whole-sellers of Y and the downstream firms are like retailers of Y. The retailers submit bids in units of X to the whole-sellers from the revenue earned by selling Y to the consumers.

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The whole-sellers offer Y to the retailers who in turn sell Y to the consumers. While whole-sellers care for both Y as well as X, the retailers care only for X. Thus, whole sellers have preferences over consumption bundles and are utility maximizers; the retailers on the other hand are profit maximizers.

The entire trading process is as follows. The whole sale price of Y being the ratio of total bids to total offers, each whole-seller earns the value of what he offers at wholesale prices; each retailer purchases the amount of Y that his bid can buy at wholesale prices; the retailers sell the Y that they obtain from the whole-sellers to the consumers at the retail price that is determined by the inverse demand function for Y of the consumers. Considering that the major objective of this paper is to introduce successive oligopolies in a pure exchange economy, we try to keep the investigation as simple as possible without sacrificing any major implication of the model. Hence we have assumed that all whole-sellers have the same Cobb-Douglas utility function defining their preferences over consumption bundles and each one of them initially own a single unit of Y. In a way this leads to a kind of symmetry on both sides of the bilateral oligopoly, since profit maximization by the retailers implies that all retailers have the same utility function which measures the satisfaction that a retailer derives from a consumption bundle simply by the amount of X that the bundle contains.

In this model, it is easy to see that there is a trivial equilibrium with no bids or offers being submitted. In addition we establish that a unique non-trivial equilibrium exists in which every trader participates in the market. Further, such equilibrium is symmetric- all whole-sellers offer the same quantity of Y and all retailers bid the same quantity of X. We also obtain some comparative static results analogous to those available in Amir and Bloch (2009). If the number of whole-sellers increases with the number of retailers remaining fixed, then the offer of each whole-seller goes up. On the other hand if the number of retailers increases with the number of whole-sellers remaining fixed then while the bid of each retailer goes down the aggregate bid goes up.

It is prudent to point out an assumption about the inverse demand function of the consumers which might limit the extent to which we may be able to vary the number of whole sellers in our model. In this paper we assume that inverse demand function of the consumers for Y is linear, downward sloping and when the market (i.e. retail) price of Y is zero, the consumers would be willing to consume more than the aggregate initial endowment of Y with the whole-sellers. We also normalize the units of measurement of Y so that the slope of the inverse demand function is one. This normalization is a standard practice in economic theory so that needless complexities may be avoided. The assumption that the demand function of the consumers is linear means that the utility function of the representative consumer is quadratic and there is a finite level of consumption at which the consumers would get satiated. Linear demand functions are commonplace in the industrial organization literature. Our assumption that the "when the market price of Y is zero, the consumers would be willing to consume more than the aggregate initial endowment of Y with the whole-sellers" means that the level of consumption at which consumers get satiated exceeds the total initial endowment of the whole sellers. This ensures that irrespective of the quantity of Y offered by the whole-sellers, the consumers would be willing to pay a positive price for what the retailers bring to the market i.e. in the relevant range of our analysis, more of Y is better as far as the consumers is concerned. This may not be necessary for our results to go through; it only saves us from needless complications given our present purpose.

THE MODEL

We consider a market with two goods i.e. money (X) and another non-monetary divisible commodity (Y). The economy has three types of agents: whole-sellers of Y, retailers of Y and consumers of Y. Each whole-seller is initially endowed with 1 unit of Y; a retailer owns nothing initially. The consumers of Y are represented by their inverse demand function for Y. Whole-sellers are indexed by s =1,…, n and retailers are indexed by b = 1,…,m. We shall sometimes refer to the collection of whole-sellers and retailers as traders.

Let x denote the quantity of money and y the quantity of Y allocated to a trader. A *consumption bundle* is a pair $(x,y) \in \mathbb{R}^2_+$.

We assume that all whole-sellers have the same utility U: $\mathbb{R}^2_+ \rightarrow \mathbb{R}$ and all retailers are profit maximizers. The retailers do not derive any benefit by consuming Y; the only good that they desire is X. Hence, the entire amount of Y that a retailer receives from the sellers in lieu of his bid, he sells to the consumers.

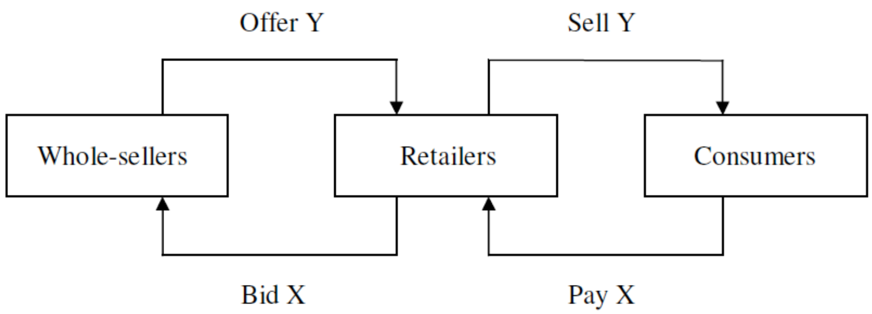

Figure 1:

Assumption 1

There exists $\alpha \in (0,1)$ such that $U(x,y) = x^{\alpha}y^{1-\alpha}$ for all $(x,y) \in \Re^2_+$.

Assumption 2:

The inverse (market) demand function, $\pi: \mathbb{R}_{++} \to \mathbb{R}$ of the consumers for Y is such that for some A > n: $\pi(y) = A - y$ for $y \le A$, $\pi(y) = 0$ for $y > A$.

Thus by assumption 2, A depends on 'n'(the number of whole-sellers).

We also make the following standard assumption in order to make bilateral oligopoly between whole-sellers and retailers meaningful.

Assumption 3

$m \ge 2$ and $n \ge 2$.

An *allocation* is a list $[(x_b, y_b)_{b=1,\ldots,m}, (x_s, y_s)_{s=1,\ldots,n}, y_c]$ such that $\sum_{b=1}^{m} x_b + \sum_{s=1}^{n} x_s = y_c\pi(y_c), \sum_{s=1}^{n} y_s + \sum_{b=1}^{m} y_b = n$ and $y_c + \sum_{b=1}^{m} y_b$ where 'x_b'(respectively 'x_s') is the quantity of X consumed by a buyer b (respectively seller s), 'y_b'(respectively 'y_s') is the quantity of Y consumed by a buyer b (respectively seller s), and 'y_c' is the total amount of Y sold by the retailers to the consumers for a retail price of $\pi(y_c)$ per unit. For the sake of simplicity we shall often denote an allocation as $[(x_b), (x_s,y_s), y_c]$.

Here we assume that each whole-seller s offers a quantity $q_s \in [0,1]$ to the retailers. The list of offers $(q_s)_{s=1,\ldots,n}$ is for the sake of simplicity denoted (q_s).We shall denote the aggregate offer of the whole-sellers $\sum_{t=1}^{n} q_t$ by Q and the aggregate offer of all whole-sellers other than s, i.e. $Q - q_s$ by Q_{-s}.

In addition to each whole-seller s offering q_s of Y each retailer b now submits a bid $g_b \ge 0$ in units of X to the whole-sellers. The bid that retailer b submits is the amount of money that b is willing to pay to the whole-sellers for Y. The list of bids $(g_b)_{b=1,\ldots,m}$ is for the sake of simplicity denoted (g_b). The total bid placed by all retailers is denoted by G and the total bid of all retailers other than b, i.e. $G - g_b$, is denoted G_{-b}.

The whole-sale price of Y if the list of offers is (q_s) and the list of bids is (g_b) is defined thus:

$P = \frac{G}{Q}$ if $Q > 0$ and $G > 0$

= 0, otherwise.

A strategy profile is a pair $[(g_b), (q_s)]$ such that for all $b = 1,\ldots,m$, g_b is retailer b's bid to the whole-sellers and for all $s = 1,\ldots,n$, q_s is the quantity of Y that the whole-seller s offers to sell to the retailers.

The outcome corresponding to the strategy profile $[(g_b), (q_s)]$ is the allocation $[(x_b, y_b), (x_s,y_s), y_c]$

such that:

(i) for all $b = 1,\ldots,m$, $(x_b, y_b) = (\frac{g_b}{P}\pi(Q) - g_b, \frac{g_b}{P})$ if $P > 0$

$= (-g_b, 0)$ if $P = 0$;

(ii) for all $s = 1,\ldots,n$, $(x_s, y_s) = (Pq_s, 1-q_s)$;

Since $[(x_b, y_b), (x_s,y_s), y_c]$ is an allocation, by definition it follows that $y_c = \sum_{b=1}^{m} y_b$.

A strategy profile $[(g_b), (q_s)]$ is said to be an *equilibrium* if:

(i) for all $b = 1,\ldots, m$: g_b maximizes $\frac{g}{g+G_{-b}}\pi(Q)$;

(ii) for all $s = 1,\ldots, n$: q_s maximizes $U(\frac{q}{q+Q_{-s}}G, 1-q)$

subject to $q \in [0,1]$.

SOME PRELIMINARY RESULTS

In this section we put together some introductory results concerning equilibrium.

Proposition 1

Let $g_b = 0$ for all b = 1,…,m and $q_s = 0$ for all s = 1,…,n. Then $[(g_b), (q_s)]$ is an equilibrium. This equilibrium is called the trivial equilibrium.

Lemma 1

Suppose $[(g_b), (q_s)]$ is an equilibrium. If G > 0 then $1 > q_s > 0$ for all s = 1,…,n. Hence, if G > 0, then Q > 0.

Proof

Suppose G > 0. First, towards a contradiction suppose $q_s = 0$ for some s. Then since $(\frac{q}{q+Q_{-s}}G, 1-q)|_{q=1/2} \in \mathbb{R}^2_{++}$, $U(\frac{q}{q+Q_{-s}}G, 1-q)|_{q=1/2} > 0 = U(0,1) = (\frac{q_s}{q_s+Q_{-s}}G, 1-q_s)$. This contradicts that $[(g_b), (q_s)]$ is an equilibrium. Thus $q_s > 0$ for all s = 1,…,n.

Now, towards a contradiction suppose $q_s = 1$ for some s. Then since $(\frac{q}{q+Q_{-s}}G, 1-q)|_{q=1/2} \in \mathbb{R}^2_{++}$, $U(\frac{q}{q+Q_{-s}}G, 1-q)|_{q=1/2} > 0 = U(\frac{q_s}{q_s+Q_{-s}}G, 0) = U(\frac{q_s}{q_s+Q_{-s}}G, 1-q_s)$. This contradicts that $[(g_b), (q_s)]$ is an equilibrium. Thus $q_s < 1$ for all s = 1,…,n. Q.E.D.

Lemma 2

Suppose $[(g_b), (q_s)]$ is an equilibrium. If Q > 0, then G > 0.

Proof:

Suppose Q > 0 and towards a contradiction suppose G = 0. Let b be a retailer. Thus, $G_{-b} = 0$. Thus $x_b = 0 = y_b$. Since $A > n \geq Q > 0$, $Q(A-Q) > 0$. Let $g \in (0, Q(A-Q))$. Thus $x_b = Q(A-Q) - g$ contradicting that $[(g_b), (q_s)]$ is an equilibrium. Thus G > 0. Q.E.D.

Combining lemmas 1 and 2 we get the following proposition.

Proposition 2

Suppose $[(g_b), (q_s)]$ is an equilibrium. Then the following three statements are equivalent.

(i) $1 > q_s > 0$ for all s = 1,…,n.

(ii) Q > 0;

(iii) G > 0.

Hence at a non-trivial equilibrium: (i) $1 > q_s > 0$ for all s = 1,…,n, and (ii) G > 0.

THE WHOLE-SELLER'S SUBPROBLEM

Given G, the whole sellers choose (q_s) such that for all s = 1,…,n: q_s maximizes $U(\frac{q}{q+Q_{-s}}G, 1-q)$ subject to $q \in [0,1]$.

Proposition 3

Given G > 0, (q_s) is an equilibrium for the whole-sellers' subproblem if and only if for all s = 1,…,n: $1 > q_s > 0$ and $\frac{GQ_{-s}}{(Q_{-s}+q_s)^2} U_x - U_y = 0$.

Proof

Suppose G > 0. First, towards a contradiction suppose $q_s = 0$ for some s. Then since $(\frac{q}{q+Q_{-s}}G, 1-q)|_{q=1/2} \in \mathbb{R}^2_{++}$, $U(\frac{q}{q+Q_{-s}}G, 1-q)|_{q=1/2} > 0 = U(0,1) = U(\frac{q_s}{q_s+Q_{-s}}G, 1-q_s)$. This contradicts that (q_s) is an equilibrium for the whole-sellers' subproblem. Thus $q_s > 0$ for all s = 1,…,n.

Now, towards a contradiction suppose $q_s = 1$ for some s. Then since $(\frac{q}{q+Q_{-s}}G, 1-q)|_{q=1/2} \in \mathbb{R}^2_{++}$, $U(\frac{q}{q+Q_{-s}}G, 1-q)|_{q=1/2} > 0 = U(\frac{q_s}{q_s+Q_{-s}}G, 0) = U(\frac{q_s}{q_s+Q_{-s}}G, 1-q_s)$. This contradicts that (q_s) is an equilibrium for the whole-sellers' subproblem. Thus $q_s < 1$ for all s = 1,…,n.

The rest of the proposition follows immediately from this. Q.E.D.

Corollary of Proposition 3

Given G > 0, (q_s) is an equilibrium for the whole-sellers' subproblem if and only if for all s = 1,…,n: $\frac{\alpha Q_{-s}}{q_s(Q_{-s}+q_s)} - \frac{1-\alpha}{1-q_s} = 0$, i.e. $(1-\alpha),q_s^2. + q_sQ_{-s} - \alpha Q_{-s} = 0$.

Proposition 4

Given G > 0, (q_s) is an equilibrium for the whole-sellers' subproblem if and only if $q_s = \frac{n-1}{n-\alpha}\alpha$ for all s =

1,…,n. Hence $Q = \frac{n-1}{n-\alpha}\alpha$. Both q_s and Q are independent of m and G and both go up as n (the number of whole-sellers) increases with m (the number of retailers) held fixed.

Proof

Given G > 0 by Proposition 3 we get that if (q_s) is an equilibrium for the whole-sellers' subproblem then $m-1 > Q_{-s} > 0$ for all s = 1,…,n.

For $Q_{-s} \in (0,m-1)$, let $q_s(Q_{-s})$ denote the unique solution of the problem

Maximize $U\left(\frac{q}{q+Q_{-s}}G, 1-q\right)$

subject to $q \in [0,1)$.

Consider the function f: $(0,m-1) \to [0,m]$ defined by $f(Q_{-s}) = Q_{-s} + q(Q_{-s})$ for all Q_{-s} belonging to (0,m-1]. The function is well defined since it is easy to show by adopting a strategy similar to the proof of Proposition 3 that $q_s(Q_{-s}) \in (0,1)$ for all $Q_{-s} \in (0,m-1)$.

We know that $(1-\alpha)q^2 + qQ_{-s} - \alpha Q_{-s} = 0$ for all $Q_{-s} \in (0,m-1)$. Differentiating this expression with respect to Q_{-s} gives us $2(1-\alpha)q(Q_{-s})\frac{dq}{dQ_{-s}} + q(Q_{-s}) + Q_{-s}\frac{dq}{dQ_{-s}} - \alpha = 0$.

Thus $\frac{dq}{dQ_{-s}} = \frac{\alpha - q(Q_{-s})}{2(1-\alpha)q(Q_{-s}) + Q_{-s}}$.

Hence $f'(Q_{-s}) = 1 + \frac{\alpha - q(Q_{-s})}{2(1-\alpha)q(Q_{-s}) + Q_{-s}}$

$= \frac{2(1-\alpha)q(Q_{-s}) + Q_{-s} + \alpha - q(Q_{-s})}{2(1-\alpha)q(Q_{-s}) + Q_{-s}}$

$= \frac{(1-\alpha)q(Q_{-s}) + \alpha(1-q(Q_{-s})) + (q(Q_{-s}) + Q_{-s})}{2(1-\alpha)q(Q_{-s}) + Q_{-s}}$ since the denominator is positive and each term in the numerator is positive.

Thus f(.) is an increasing function of Q_{-s}.

Towards a contradiction suppose (q_s) is an equilibrium for the whole-sellers' subproblem with $q_s \neq q_t$ for some $s,t \in \{1,…,n\}$. Thus $Q_{-s} \neq Q_{-t}$ although $f(Q_{-s}) = f(Q_{-t})$. This contradicts that f is strictly increasing.

Hence $q_s = q_t$ for all $s,t \in \{1,…,n\}$.

Let Q denote the aggregate offers by the whole-sellers at the equilibrium. Then $q_s = \frac{Q}{n}$ for all s = 1,…,n.

Thus, $(1-\alpha)(\frac{Q}{n})^2 + \frac{Q}{n}\frac{(n-1)Q}{n} - \alpha\frac{(n-1)Q}{n} = 0$

or $(1-\alpha)\frac{Q}{n} + \frac{(n-1)Q}{n} = \alpha(n-1)$.

Hence $\frac{Q}{n} = \frac{n-1}{n-\alpha}\alpha$ and so $q_s = \frac{n-1}{n-\alpha}\alpha$ for s = 1,…,n.

Conversely suppose $q_s = \frac{n-1}{n-\alpha}\alpha$ for s = 1,…,n. Then for s=1,…n, q_s maximizes $U(\frac{m(1-\alpha)}{Q_{-s}+q}q, 1-q)$ subject to $q \in [0,1]$ if and only if $\frac{\alpha Q_{-s}(Q_{-s}+q_s)}{q_s} - \frac{(1-\alpha)(Q_{-s}+q_s)^2}{1-q_s} = 0$, where $Q_{-s} = \frac{(n-1)^2}{n-\alpha}\alpha$

Thus q_s maximizes $U(\frac{q}{q+Q_{-s}}G, 1-q)$ subject to $q \in [0,1]$ if and only if $\frac{\alpha Q_{-s}}{q_s} - \frac{(1-\alpha)(Q_{-s}+q_s)}{1-q_s} = 0$.

It is now easy to see that $q_s = \frac{n-1}{n-\alpha}\alpha$ does indeed satisfy the preceding equation. Thus (q_s) is a equilibrium for the whole-sellers' subproblem given G.

Since $0 < \alpha < 1$, $\frac{n-1}{n-\alpha}$ increases as 'n' increases and hence $q_s = \frac{n-1}{n-\alpha}\alpha$ increases as n increases. Further, $Q = n\alpha\frac{n-1}{n-\alpha}$ increases as 'n' increases. Q.E.D.

THE RETAILER'S SUB-PROBLEM

Given $Q = n\alpha\frac{n-1}{n-\alpha}$, the retailers choose (g_b) such that for all b = 1,…,m: g_b maximizes $[\frac{g}{g+G_{-b}}Q(A-Q) - g]$ subject to $\frac{g}{g+G_{-b}}Q(A-Q) \geq g \geq 0$. Notice that if g = 0, then retailer b's profit is equal to zero. Hence retailer b has no incentive to choose a g such that $g > \frac{g}{g+G_{-b}}Q(A-Q)$.

Hence retailer b's problem reduces to maximizing $[\frac{g}{g+G_{-b}}Q(A-Q) - g]$ subject to $g \geq 0$.

Proposition 5

(g_b) is an equilibrium for the retailers' subproblem if and only if for all b = 1,…,m: $\frac{G_{-b}}{G^2}\mathrm{Q}(\mathrm{A}-\mathrm{Q}) \leq 1$, with strict inequality only if $g_b = 0$.

Proposition 6

(g_b) is an equilibrium for the retailers' subproblem with $g_b > 0$ for all b = 1,…,m, if and only if $\mathrm{g_b} = \frac{(m-1)Q(\mathrm{A}-Q)}{m^2}$ for all b = 1,…, m. Thus $\mathrm{G} = \frac{(m-1)Q(\mathrm{A}-Q)}{m}$ and G goes up as m increases provided n (the number of whole-sellers) remains constant. However as m goes up with n remaining fixed, g_b decreases for all b = 1,…, m.

Proof

First notice that if $G_{-b} = 0$, then the problem maximize $[\frac{g}{g+G_{-b}}\mathrm{Q}(\mathrm{A}-\mathrm{Q})-\mathrm{g}]$ subject to $g \geq 0$ has no solution. Hence we may as well suppose $G_{-b} > 0$.

For $G_{-b} > 0$, let $g_b(G_{-b})$ denote the unique solution of the problem maximize $[\frac{g}{g+G_{-b}}\mathrm{Q}(\mathrm{A}-\mathrm{Q})-\mathrm{g}]$ subject to $g \geq 0$.

Then $g_b(G_{-b}) > 0$ if and only if $\frac{G_{-b}}{(g+G_{-b})^2}Q(A-Q)\big|g=0 > 1$ i.e. $Q(A-Q) > G_{-b}$.

Consider the function F: $\mathbb{R}_{++} \to \mathbb{R}_{++}$ defined by $F(G_{-b}) = G_{-b} + g(G_{-b})$ for all G_{-b} belonging to (0, Q(A-Q)).

We know that $(g(G_{-b}) + G_{-b})^2 = G_{-b}Q(A-Q)$ for all $G_{-b} \in (0, Q(A-Q))$. Differentiating this expression with respect to G_{-b} gives us $2(g(G_{-b})+G_{-b})$ $(\frac{dg}{dG_{-b}}+1) = \mathrm{Q}(\mathrm{A}-\mathrm{Q})$.

Now $F'(G_{-b}) = \frac{dg}{dG_{-b}}+1$ and $F(G_{-b}) = g(G_{-b}) + G_{-b} > 0$ for all $G_{-b} \in (0, Q(A-Q))$. Thus $F'(G_{-b}) = \frac{Q(A-Q)}{2F(G_{-b})} > 0$.

Thus F is an increasing function of G_{-b} on the domain (0, Q(A-Q)).

Towards a contradiction suppose (g_b) is an equilibrium for the retailers' subproblem, with $g_a \neq g_b$ for some $a,b \in \{1,…,m\}$. Thus $G_{-a} \neq G_{-b}$ although $F(G_{-a}) = F(G_{-b})$. This contradicts that F is strictly increasing.

Hence $g_a = g_b$ for all $a,b \in \{1,…,m\}$.

Let G denote the aggregate bids by the retailers at the equilibrium for the retailers subproblem.

Then $g_b = \frac{G}{m}$ for all b = 1,…,m.

Thus, $G^2 = (m-1)\frac{G}{m}Q(A-Q)$

or $G = (m-1)Q\frac{A-Q}{m}$.

Hence $\mathrm{g_b} = \frac{(m-1)Q(A-Q)}{m^2}$ for all b = 1,…,m.

The converse is easily established.

Since $\frac{m-1}{m}$ increases as m increases, we get that G increases as 'm' increases provided 'n' is held fixed.

Now $\frac{m-1}{m^2} - \frac{m}{(m+1)^2} = \frac{(m^2-1)(m+1)-m^3}{m^2(m+1)^2}$

$= \frac{m^2-m-1}{m^2(m+1)^2} = \frac{m(m-1)-1}{m^2(m+1)^2} > 0$ for all $m \geq 2$.

Thus as m increases g_b decreases provided n remains fixed. Q.E.D.

EXISTENCE OF NON-TRIVIAL EQUILIBRIUM

It is clear that a non-trivial equilibrium can be obtained only when the offers are as stated in Proposition 4 and the bids are as stated in Proposition 6.

Theorem 1

There exists a unique non-trivial equilibrium [(g_b), (q_s)] with $g_b > 0$ for all b = 1,…, m. At such an equilibrium $\mathrm{q_s} = \frac{n-1}{n-\alpha}$ α for all s = 1,…,n and $\mathrm{g_b} = \frac{(m-1)Q(A-Q)}{m^2}$ for all b = 1,…, m, where Q = $n\alpha\frac{n-1}{n-\alpha}$. Thus Q = $n\frac{n-1}{n-\alpha}$ α and $\mathrm{G} = \frac{(m-1)Q(A-Q)}{m}$. The whole sale price of Y at this equilibrium is $\frac{(m-1)(A-Q)}{m}$ and the aggregate profits of the retailers is $\frac{Q(A-Q)}{m}$.

An equilibrium $[(g_b, r_b),(q_s)]$ is said to be *symmetric* if there exists g^o, r^o, q^o such that for all b = 1,…, m, $(g_b, r_b) = (g^o, r^o)$ and for all s = 1,…, n, $q_s = q^o$.

Corollary of Theorem 1

There exists a unique non-trivial equilibrium $[(g_b), (q_s)]$ with $g_b > 0$ for all b = 1,…, m, which is also a symmetric equilibrium.

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