

# A Power Flow Method for Radial Distribution Feeders with DER Penetration

Panagiotis M. Anagnostopoulos\* and Stavros A. Papathanassiou

School of Electrical and Computer Engineering, National Technical University of Athens (NTUA), 9 Iroon Polytechniou str., 15780 Athens, Greece

**Abstract:** This paper presents a novel power flow method suitable for radial distribution feeders, which consists a modification of the simplified power flow concept known as the *DistFlow* method, already available in the literature. The proposed method relies upon a differentiated manipulation of power losses, which are taken into account in voltage calculations, unlike other simplified methods, where losses are totally neglected. As a result, calculation accuracy is greatly improved, in terms of node voltages, losses and overall active & reactive power flows. In addition, the proposed method is non-iterative and entirely linear, being easily implementable and fast in execution. The method is particularly suited for feeders with a high penetration of Distributed Energy Resources (DER), providing results that closely match those of a full non-linear power flow and are considerably more accurate than the traditional linearized distribution power flow methods, without any increase in computational burden. The new method is applied to a variety of case studies in the paper, to demonstrate its accuracy and effectiveness, comparing its performance with the simplified (linearized) *DistFlow* and a conventional non-linear power flow method.

**Keywords:** Distribution power flow, voltage calculation, power losses calculation, distributed energy resources, *DistFlow*, *ModDistFlow*.

## 1. INTRODUCTION

Load flow analysis is a fundamental tool for design and operation of distribution networks [1-5], whose importance is amplified by the growing attention on distribution systems, the ever increasing penetration and significance of Distributed Energy Resources (DER) [4-6], the development of Distribution Management Systems (DMS) [7, 8] the implementation of new and advanced functionalities such as demand response and the continuously increasing load demand [9]. At the same time, the size and complexity of the systems are also increasing [2, 4, 9], rendering the task of analysis more difficult and cumbersome, especially if the non-linear nature of power flow is taken into account. For these reasons, fast, accurate, robust, and computationally efficient distribution power flow algorithms become all the more significant [3, 7, 9].

Conventional load flow techniques (e.g. Gauss Seidel, Newton Raphson, Decoupled and Fast Decoupled Load Flow [1-3, 9-16]) are known to be unsuitable for distribution networks, in terms of efficiency, performance and robustness [1-3, 9-21]. Hence, a variety of power flow algorithms [1-24] have been developed so far for radial or meshed distribution networks, different load models, varying levels of convex relaxation etc. The vast majority of these

methods are either iterative or based on non-linear equations and rather complicated mathematical or numerical approaches, which renders them inefficient and cumbersome to apply. A notable exception relying on basic circuit theory and simple voltage equations is the *DistFlow* power flow method, initially presented in [25-27], which is suitable for radial distribution networks. It is based on a set of simple, recursive equations, where no trigonometric terms are used. Dropping the quadratic terms that represent losses and making certain approximations in voltage calculation, a simplified (linearized) form of *DistFlow* is derived. A comparison of two *DistFlow* algorithm variants is presented in [16] for different loading conditions and feeder R/X ratios. In [12], three algorithms (decoupled, fast decoupled & very fast decoupled distribution load-flow algorithms) have been proposed for radial distribution networks, based on the *DistFlow* methodology.

Although the *DistFlow* equations have been widely used in the literature, in their original form or with minor modifications [15, 23, 24, 28-36], they still suffer from high errors in power flows, losses and voltage calculation in networks with heavy load and/or high DER penetration, as a result of the fundamental assumption of neglecting active and reactive power losses. Inaccuracies become more pronounced when operating power factors deviate from unity (e.g. in networks where DER participate in voltage regulation), as well as in feeders with high R/X ratios or high overall impedance.

Address correspondence to this article at the School of Electrical and Computer Engineering, National Technical University of Athens (NTUA), 9 Iroon Polytechniou str., 15780 Athens, Greece; Tel: +30 210 7723658; Fax: +30 210 7723593; E-mail: panagiotis\_anagnostopoulos@central.ntua.gr

In this paper, a novel distribution power flow method is introduced, based on a modification of the standard, simplified *DistFlow* equations, so as to maintain active and reactive power losses and properly incorporate them in the algorithm equations. The proposed method, referred to in the following as the *ModDistFlow* method, is linear, non-iterative, easy to implement, fast and particularly robust, presenting enhanced accuracy in power flow, losses and voltage calculations, compared to the linearized *DistFlow* method. To demonstrate its performance, a variety of benchmarking scenarios are presented in the paper, addressing different loading conditions, DER penetration levels, network characteristics (R/X ratios) etc., including application to a real world MV network. In all cases, the proposed method is compared to the simplified (linearized) *DistFlow* and to the full non-linear power flow.

The paper is organized as follows. Section 2 describes the standard simplified *DistFlow* method. The proposed method is introduced in Section 3. The evaluation of the method using a simple reference network is presented in Section 4, while in Section 5 a real MV rural feeder is used as a case study, whose parameters are provided in the Appendix. Concluding remarks are summarized in Section 6.

## 2. THE *DISTFLOW* AND *SIMDISTFLOW* POWER FLOW METHODS

The widely used *DistFlow* simplified power flow method for radial distribution networks is based on the work of Baran and Wu, [25-27]. Its primary equations are outlined in the following, incorporating modifications introduced since its introduction.

With reference to Figure 1,  $V_0$  represents the voltage at feeder departure. For MV feeders this typically is the voltage of the substation MV busbars, regulated by the On-Load Tap Changer (OLTC) of the HV/MV transformer and voltage regulator relay, and therefore it is assumed to be known. Lines are represented by their series impedance  $z_j=r_j+jx_j$ , ignoring shunt capacitance. All loads are considered constant

power.  $S_{j,s}$  is the “sending” complex power flow of branch  $j$ , i.e. the left-hand side complex power flow entering the branch at the side of node  $j$ , while  $S_{j,r}$  is the “receiving” complex power flow of branch  $j$ , i.e. the right-hand side complex power flow leaving the branch, at the side of node  $j+1$ .

Given the voltage of node  $j$  and the power flow at the left-hand side of branch  $j$  (“sending” power), the voltage of node  $j+1$  and the sending power of branch  $j+1$  can then be calculated based on the following equations:

$$S_{j+1,s} = S_{j,s} - S_{loss_j} - s_{load_{j+1}} = S_{j,s} - z_j \frac{|S_{j,s}|^2}{V_j^2} - s_{load_{j+1}} \quad (1)$$

$$V_{j+1} = V_j - z_j \mathbf{I}_j = V_j - z_j \frac{S_{j,s}^*}{V_j} \quad (2)$$

Using  $V_j$  as the reference complex vector, Eqs. (1) and (2) for branch  $j$  yield the following real equations:

$$P_{j+1,s} = P_{j,s} - r_j \frac{P_{j,s}^2 + Q_{j,s}^2}{V_j^2} - P_{load_{j+1}} \quad (3.i)$$

$$Q_{j+1,s} = Q_{j,s} - x_j \frac{P_{j,s}^2 + Q_{j,s}^2}{V_j^2} - q_{load_{j+1}} \quad (3.ii)$$

$$V_{j+1}^2 = V_j^2 - 2(r_j P_{j,s} + x_j Q_{j,s}) + (r_j^2 + x_j^2) \frac{P_{j,s}^2 + Q_{j,s}^2}{V_j^2} \quad (3.iii)$$

Eqs. (3.i)-(3.iii) are known as the forward *DistFlow* branch equations [27], employing only quantities at the left end of each branch (sending).

Assuming that node voltages are close to nominal, a reasonable approximation when node voltage deviations are small [32-34] leads to the following Eq. (4):

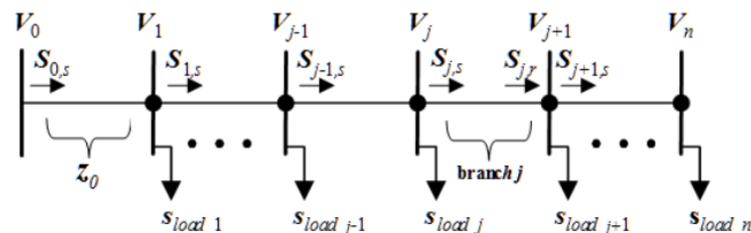


Figure 1: Notation employed in the SimDistFlow method.

$$V_j - V_{nom} \approx 0 \Rightarrow (V_j - V_{nom})^2 \approx 0 \Rightarrow V_j^2 \approx 2V_j V_{nom} - V_{nom}^2 \quad (4)$$

Rewriting Eq. (4) for node  $j+1$  and subtracting from Eq. (4) yields:

$$V_{j+1}^2 - V_j^2 \approx 2V_{nom} (V_{j+1} - V_j) \quad (5)$$

Eq. (5) can be substituted in Eq. (3.iii), while the non-linear terms in Eqs. (3.i)-(3.iii) represent active and reactive power losses and can be neglected. Based on these assumptions, the following simplified *DistFlow* equations [27, 33] (hereinafter referred to as the *SimDistFlow* method) are derived:

$$P_{j+1,s} = P_{j,s} - p_{load\_j+1} \quad (6.i)$$

$$Q_{j+1,s} = Q_{j,s} - q_{load\_j+1} \quad (6.ii)$$

$$V_{j+1} = V_j - \frac{r_j P_{j,s} + x_j Q_{j,s}}{V_{nom}} \quad (6.iii)$$

$$P_{loss\_j} = r_j \frac{P_{j,s}^2 + Q_{j,s}^2}{V_{nom}^2} \quad (6.iv)$$

$$Q_{loss\_j} = x_j \frac{P_{j,s}^2 + Q_{j,s}^2}{V_{nom}^2} \quad (6.v)$$

The Eqs (6.i)-(6.v) are solved using the following boundary conditions for node 0 at feeder departure and node  $n$  at feeder end:

- $V_0$  is considered to be known
- $P_{n,s} = Q_{n,s} = 0$

A slightly modified version of these equations is presented in [31, 32, 34], where  $V_0$  is used instead of  $V_{nom}$ .

### 3. THE PROPOSED MODDISTFLOW METHOD

As mentioned in the previous section, *SimDistFlow* employs quantities at the left end of each branch

(sending), which are not readily obtainable from node loads, unless active and reactive losses are ignored. In the method proposed in this paper, this fundamental shortcoming of *SimDistFlow* is overcome using the power flows at the right end of each branch (receiving) and incorporating power losses in the voltage calculations in a novel and intuitive manner.

With reference to Figure 2 and using  $V_{j+1}$  as the reference complex vector:

$$S_{j,r} = S_{j+1,s} + s_{load\_j+1} \Rightarrow \begin{cases} P_{j,r} = P_{j+1,s} + p_{load\_j+1} \\ Q_{j,r} = Q_{j+1,s} + q_{load\_j+1} \end{cases} \quad (7)$$

$$V_j = V_{j+1} + z_j I_j = V_{j+1} + z_j \frac{S_{j,r}^*}{V_{j+1}} \Rightarrow V_j^2 = V_{j+1}^2 + 2(r_j P_{j,r} + x_j Q_{j,r}) + (r_j^2 + x_j^2) \frac{P_{j,r}^2 + Q_{j,r}^2}{V_{j+1}^2} \quad (8)$$

Eq. (8) is differentiated from Eq. (2) in the selection of the reference complex vector, which now is  $V_{j+1}$  instead of  $V_j$ , facilitating implementation of the method.

Using Eqs. (7) and (8), Eqs. (3.i)-(3.iii) can be rewritten as follows:

$$P_{j-1,r} = P_{j,r} + r_j \frac{P_{j,r}^2 + Q_{j,r}^2}{V_{j+1}^2} + p_{load\_j} \quad (9.i)$$

$$Q_{j-1,r} = Q_{j,r} + x_j \frac{P_{j,r}^2 + Q_{j,r}^2}{V_{j+1}^2} + q_{load\_j} \quad (9.ii)$$

$$V_{j+1}^2 = V_j^2 - 2(r_j P_{j,r} + x_j Q_{j,r}) - (r_j^2 + x_j^2) \frac{P_{j,r}^2 + Q_{j,r}^2}{V_{j+1}^2} \quad (9.iii)$$

Eqs. (9.i)-(9.iii) are known as the *backward DistFlow* branch equations, first introduced in [27] using different notation and reference voltage.

Eq. (5) can then be employed to simplify Eq. (9.iii) as follows:

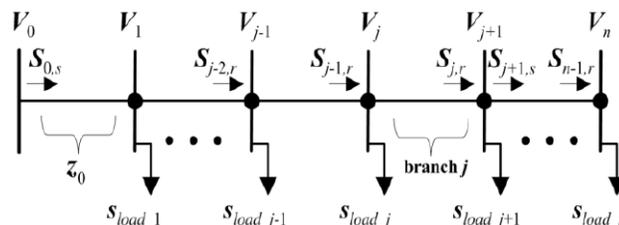


Figure 2: Notation employed in the ModDistFlow method.

$$V_{j+1} = V_j - \frac{r_j P_{j,r} + x_j Q_{j,r}}{V_{nom}} - \frac{1}{2V_{nom}} (r_j^2 + x_j^2) \frac{P_{j,r}^2 + Q_{j,r}^2}{V_{nom}^2} \quad (10)$$

where the assumption  $V_{j+1} \approx V_{nom}$  has been used in the denominator, as it is common in the literature, in order to transform a hard non-linear term to a linear one. The impact of this assumption on the accuracy of calculations is only moderate, if an effective voltage regulation policy is implemented and node voltages deviate reasonably from nominal.

The quadratic term in Eq. (10), often ignored in the literature, can be expressed in terms of the losses on branch  $j$ :

$$\frac{1}{2V_{nom}} (r_j^2 + x_j^2) \frac{P_{j,r}^2 + Q_{j,r}^2}{V_{nom}^2} = \frac{r_j}{2V_{nom}} r_j \frac{P_{j,r}^2 + Q_{j,r}^2}{V_{nom}^2} + \frac{x_j}{2V_{nom}} x_j \frac{P_{j,r}^2 + Q_{j,r}^2}{V_{nom}^2} = \frac{r_j}{2} \frac{P_{loss\_j}}{V_{nom}} + \frac{x_j}{2} \frac{Q_{loss\_j}}{V_{nom}} \quad (11)$$

Eq. (11) implies that active and reactive losses on branch  $j$  effectively cause the same voltage drop as an equivalent load would produce on a branch having half the impedance of the actual branch  $j$ . Hence, the effect of losses on voltage drop calculations can be accounted for by introducing a fictitious node at the middle of each branch, where a virtual load is connected whose complex power is equal to the active and reactive losses of the branch (Figure 3).

Based on the above, the final set of equations of the proposed method, referred to as the *ModDistFlow* method, is the following. Quantities involved and notation used are clarified in Figure 3:

$$P_{j-1,r} = P_{j,r} + P_{loss\_j} + p_{load\_j} \quad (12.i)$$

$$Q_{j-1,r} = Q_{j,r} + Q_{loss\_j} + q_{load\_j} \quad (12.ii)$$

$$V_{j+1} = V_j - \frac{r_j P_{j,r} + x_j Q_{j,r}}{V_{nom}} - \frac{r_j}{2} \frac{P_{loss\_j}}{V_{nom}} - \frac{x_j}{2} \frac{Q_{loss\_j}}{V_{nom}} \quad (12.iii)$$

$$P_{loss\_j} = r_j \frac{P_{j,r}^2 + Q_{j,r}^2}{V_{nom}^2} \quad (12.iv)$$

$$Q_{loss\_j} = x_j \frac{P_{j,r}^2 + Q_{j,r}^2}{V_{nom}^2} \quad (12.v)$$

Eqs. (12) show that losses are properly accounted for in the equilibrium of active and reactive powers for

each node, as well as in voltage calculations. Notably, the fictitious nodes in Figure 3 offer a physical explanation for the effect of losses, but they are not necessary for the implementation of the *ModDistFlow* method.

Eqs. (12.i)-(12.v) are solved using the following boundary conditions:

- i. At the end of the feeder:

$$P_{n-1,r} = p_{load\_n} \quad (13.i)$$

$$Q_{n-1,r} = q_{load\_n} \quad (13.ii)$$

- ii. At feeder departure (bus 0 in Figure 2 and Figure 3), corresponding to the substation MV busbars, the voltage is determined by the OLTC regulator and therefore it is considered to be known:

$$V_0 = V_{OLTC} \quad (13.iii)$$

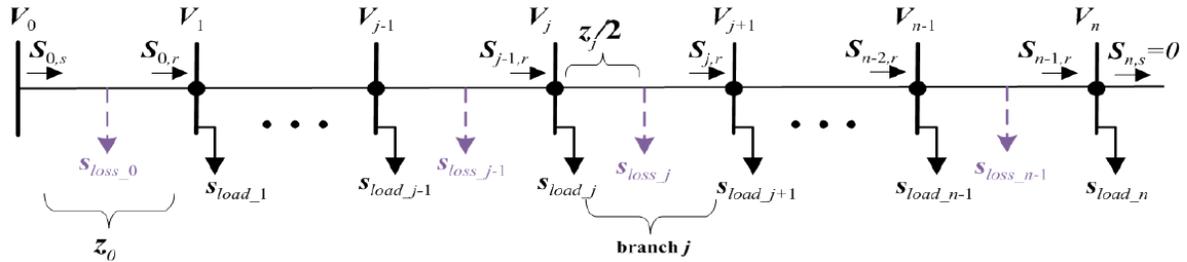
The equations of the proposed power flow method are solved in a mixed forward and backward manner:

- Power flows and losses are first determined in a backward manner. Starting with the boundary conditions (13.i)-(13.ii), the complex power flow at the last branch (branch  $n-1$ ) is first calculated. Eqs. (12.i), (12.ii), (12.iv), (12.v) are then recursively applied to backtrack towards the source (branch 0), determining active and reactive power flows and losses for every branch of the feeder.
- Having determined all power quantities in the right hand side of Eq. (12.iii), for every branch, node voltages are then calculated in a forward manner successively applying Eq. (12.iii). The boundary condition of Eq. (13.iii) provides the voltage at the source node 0.

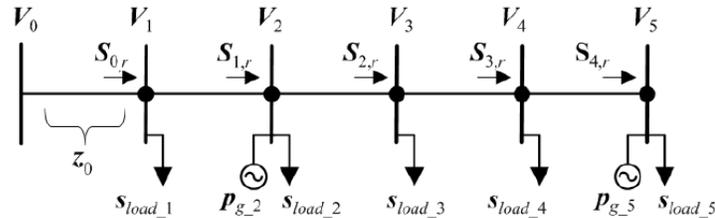
The proposed method is directly applicable to networks including DER, simply replacing  $p_{load\_j}$  and  $q_{load\_j}$  with the net active and reactive power demands at each node (local consumption minus local generation).

#### 4. EVALUATION OF THE PROPOSED POWER FLOW METHOD IN A SIMPLIFIED DISTRIBUTION NETWORK

This section demonstrates the enhanced accuracy characteristics of the proposed *ModDistFlow* method,



**Figure 3:** MV feeder equivalent in the ModDistFlow method, incorporating fictitious nodes at mid-branch positions to account for the effect of losses.



**Figure 4:** Simplified case study of a MV feeder.

as compared to *SimDistFlow*, using the simplified MV feeder of Figure 4 as a case study. Feeder characteristics are summarized in Appendix (see Table 3).

The analysis encompasses different loading conditions, DER penetration levels, feeder R/X ratios and DER power factor regulation alternatives. To evaluate the accuracy of the methods, the full non-linear power flow on the feeder (hereinafter referred to as the *Full Power Flow*) is used as a benchmark. The *Full Power Flow* has been performed using the commercial load flow software PSS-Adept.

#### 4.1. Implementation for Different Feeder Loading Conditions and DER Penetration Levels

Two case studies are analyzed, corresponding to different loading conditions of the feeder:

- High feeder load: 9 MW
- Low feeder load: 3 MW

Figure 5 shows the voltage profile along the feeder for two extreme scenarios (high load conditions and no DER connected and light load conditions and 9 MW DER connected), as calculated by *ModDistFlow*, *SimDistFlow* and the *Full Power Flow*, as well as the relative voltage error of the two former methods compared to the latter for both scenarios.

*ModDistFlow* clearly proves substantially more accurate than *SimDistFlow*, especially at distant nodes

where the standard *SimDistFlow* exhibits highest errors. Overall, the results obtained by *ModDistFlow* essentially coincide with those of the accurate Full Power Flow.

Figure 6 demonstrates voltage calculation accuracy (mean absolute relative voltage error) of *ModDistFlow* and *SimDistFlow*, assuming different DER capacities, both for high and low feeder load conditions.

In all cases, DER units operate at full output power with unity power factor. With either method, calculation error is minimized when DER output matches the load of the feeder, as power flows on feeder branches then reach a minimum, whereas highest errors are noted when DER and load levels differ significantly. Again, the *ModDistFlow* method achieves superior accuracy (error reduced by at least one order of magnitude).

Besides improvements in voltage calculation, *ModDistFlow* permits a more accurate evaluation of active and reactive power losses along the feeder, as demonstrated in Table 1 and 2. At high load conditions, the proposed method proves to be 10 times more accurate, with the absolute error remaining below 1%. At low feeder loads, *ModDistFlow* remains more accurate but by a lower margin.

#### 4.2. Additional Investigations

The effectiveness of the *ModDistFlow* method increases when applied to MV feeders with increased R/X ratios (conductors of lower cross section or

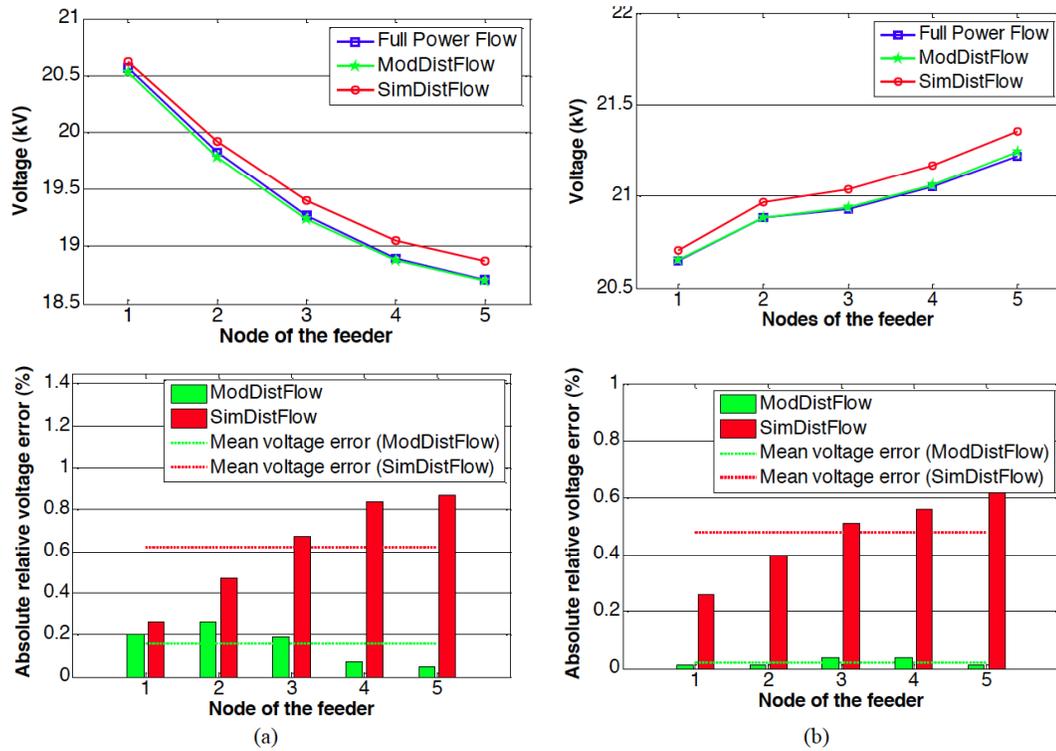


Figure 5: Voltage profile along the feeder and absolute relative voltage error (% , w.r.t. Full Power Flow solution) a) High load, no DER and b) Light load, 9 MW DER.

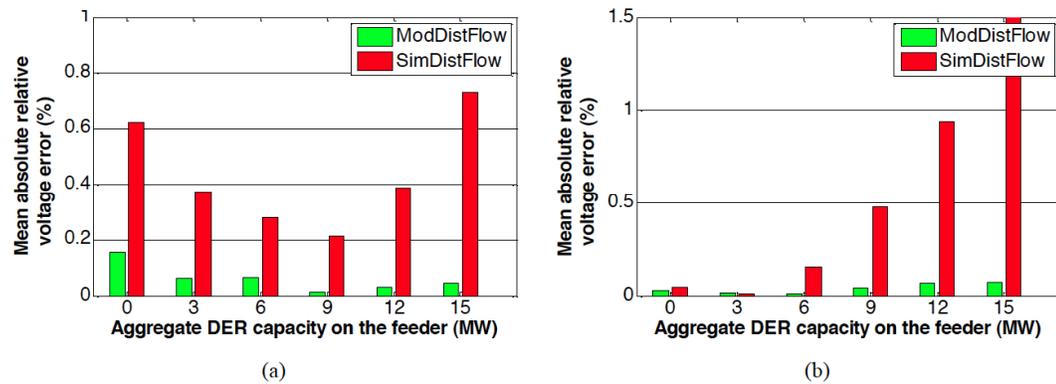


Figure 6: Voltage calculation error of ModDistFlow and SimDistFlow vs installed DER capacity on the feeder for a) high and b) low feeder load conditions.

Table 1: Comparison of Active and Reactive Power Losses Obtained by the Distribution Power Flow Algorithms – High Load, no DER Connected to the Feeder

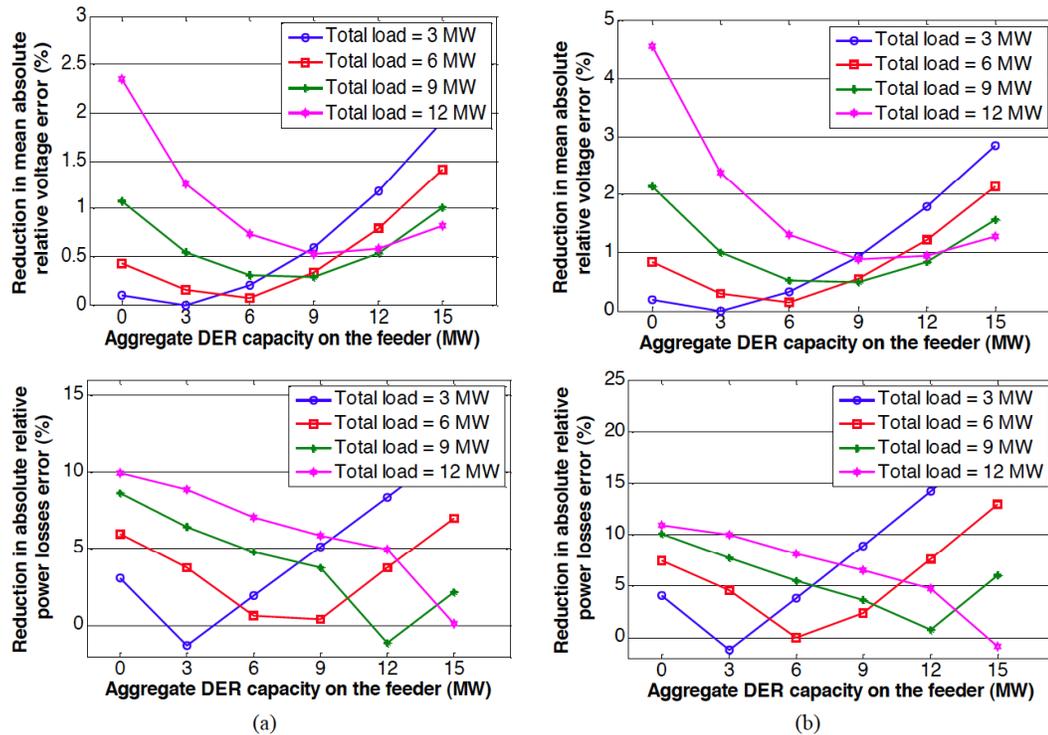
	Full Power Flow	ModDistFlow		SimDistFlow	
		losses	error (%)	losses	error (%)
Active power losses (kW)	655	651	0.7%	605	7.7%
Reactive power losses (kVAr)	1042	1035	0.7%	962	7.7%

cables). The reduction in voltage calculation error by applying *ModDistFlow* instead of *SimDistFlow* is shown in Figure 7 (mean error for all 5 nodes), as a function of

the total DER capacity on the feeder and for different loading levels, assuming R/X ratios of 1.0 and 1.5 (instead of 0.63 for the base case feeder). Higher R/X

**Table 2: Comparison of Active and Reactive Power Losses Obtained by the Distribution Power Flow Algorithms– Low Load, 9 MW of DER Connected to the Feeder**

	Full Power Flow	ModDistFlow		SimDistFlow	
		losses	error (%)	losses	error (%)
Active power losses (kW)	291	316	8.6%	323	11.2%
Reactive power losses(kVAr)	463	503	8.6%	514	11.2%



**Figure 7:** Reduction in mean voltage calculation error and active power losses ( $\Delta e = e_{SimDistFlow} - e_{ModDistFlow}$ ), by applying ModDistFlow compared to SimDistFlow. (a) MV feeder R/X = 1.0 and (b) MV feeder R/X = 1.5.

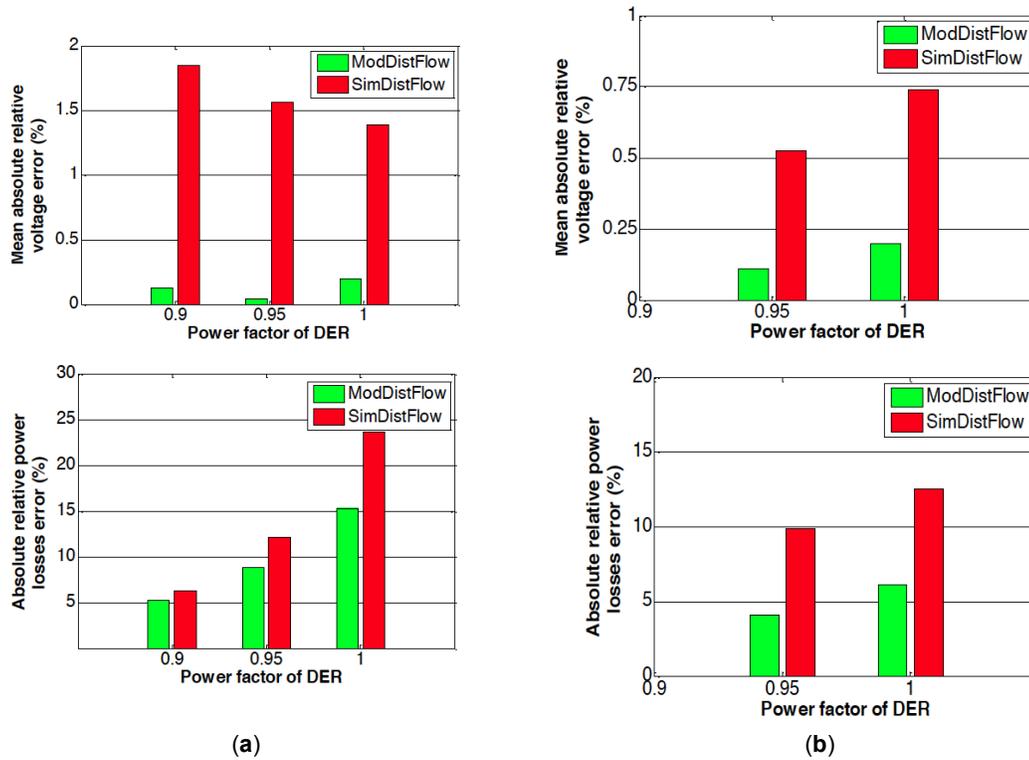
ratios lead to increased voltage calculation errors using the standard *SimDistFlow* method, as well as to greater accuracy enhancement by applying *ModDistFlow*. Similar remarks hold for the estimation of power losses on the feeder. Gains in accuracy by applying the proposed method, presented in Figure 7 become more significant at high R/X ratios, where the level of losses is increased. It is obvious in the figures on the left there is a similar monotony for several load levels along the aggregate DER capacity for voltage and power losses calculation. The same happens also in the figures on the right where a higher R/X ratio is applied. Comparing the left to the right figures, it is evident that the accuracy by implementing the *ModDistFlow* is much higher (almost 1 time and 0,7 time in voltage and power losses calculation respectively).

In all previous analysis, DER units were assumed to operate at unity power factor (zero reactive output

power). In Figure 8, indicative results for voltage and power losses calculation are provided when the power factor of DER units varies between 0.9 inductive (VAR consumption) to 1, for a reference case study of low feeder load conditions. In the same figure, similar results are presented for 0.95 capacitive (VAR production) and unity power factor operation, for a reference case study of high feeder load conditions; capacitive power factor operation may not be realistic, but it has been included only for the sake of completeness of the analysis. As it is observed, node voltages and losses calculation accuracy is enhanced by applying the *ModDistFlow* method in all cases.

**5. IMPLEMENTATION IN A REAL MV NETWORK**

To assess the performance of the *ModDistFlow* method in more realistic conditions, a real world MV feeder with substantial RES penetration is examined,



**Figure 8:** Voltage calculation error and Active power losses calculation error of ModDistFlow and SimDistFlow vs. DER power factor. a) Total load 3 MW, 12 MW DER, feeder R/X=1, inductive power factor and b) Total load 9 MW, 3 MW DER, feeder R/X=1, capacitive power factor.

located on Rhodes island, Greece. Its main characteristics are given in the Appendix (see Table 4). The feeder is shown in Figure 9 while load and DER distribution along the feeder is shown in Figure 10.

Figure 11 depicts the calculated voltage profile and respective error using *ModDistFlow*, *SimDistFlow* and the *Full Power Flow* methods for operation of the feeder at minimum load and maximum DER output. Both distribution power flow methods prove sufficiently accurate, with an average absolute error around 0.1-0.2%, with *ModDistFlow* exhibiting again improved accuracy characteristics. As for power losses, *ModDistFlow* presents an error of 7.8%, compared to 10.1% for the *SimDistFlow* method.

The differences between the two methods are not spectacular in this case, as the installed DER capacity on the feeder is relatively low at 5 MW, hence power flows and therefore losses are not significant enough to play a crucial role in voltage calculations, as in other cases examined in the previous section. At higher DER penetration levels errors increase, as well as the improvement achieved by the proposed *ModDistFlow* method. It is reasonable to state that if the power flow is very low then there is almost no margin for presenting inaccuracy by implementing any method. But what increases the merit of *ModDistFlow* is that the power flows in the future are expected to be higher taking into account the increase of both energy consumption as well as energy generation at least

**Table 3: Feeder Characteristics of the Simplified Case Study Feeder of Figure 4.**

Characteristic	Value
Feeder nominal voltage	20 kV
Feeder length	25 km
Feeder impedance	$Z=0.22+j0.35\Omega/\text{km}$ ( $R/X=0.63$ )
Nodes	1 node per 5 km
Load allocation	Equally distributed total feeder nodes
DER	Connected at nodes 2 and 5

Table 4: Main Characteristics of the MV Distribution Network

Characteristic	Value
Feeder nominal voltage	20 kV
Feeder length	38 km
Conductor types	
Overhead 3x95 mm <sup>2</sup> ACSR	$I_{thermal} = 448 \text{ A} / r+jx = 0.215 + j0.334 \text{ } \Omega/\text{km}$
Overhead 3x95 mm <sup>2</sup> Cu	$I_{thermal} = 352 \text{ A} / r+jx = 0.220 + j0.358 \text{ } \Omega/\text{km}$
Max/min load	4.5/0.9 MW
Installed RES capacity	5 MW
PV plants	20 x 0.1 MW
Wind farm	3 MW

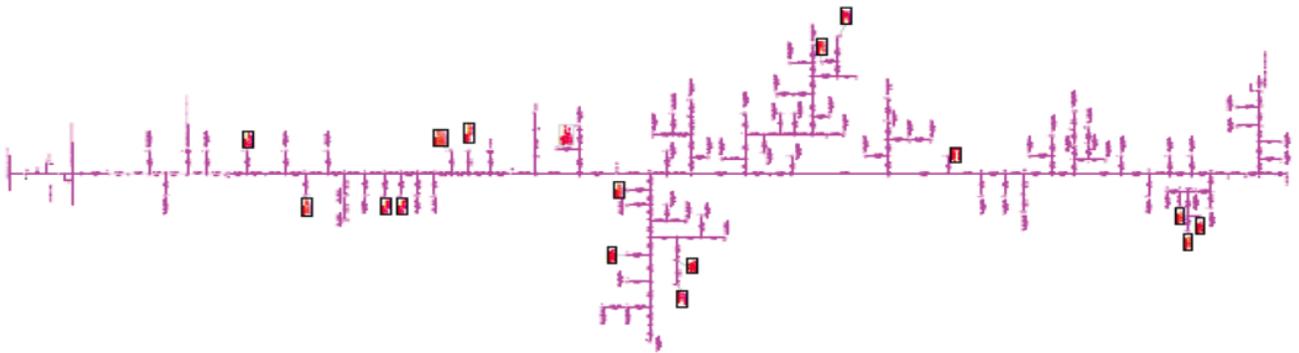


Figure 9: Real-world case study MV distribution feeder.

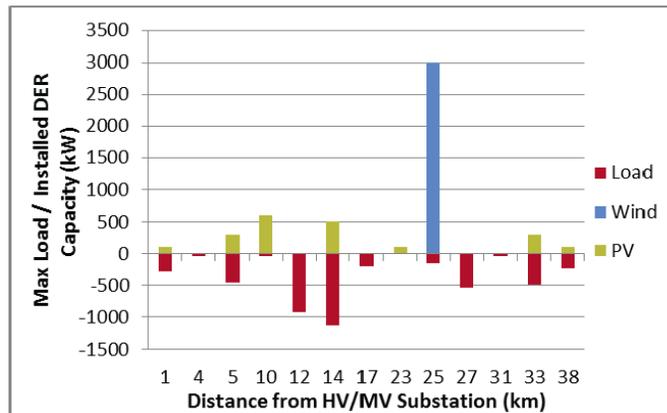


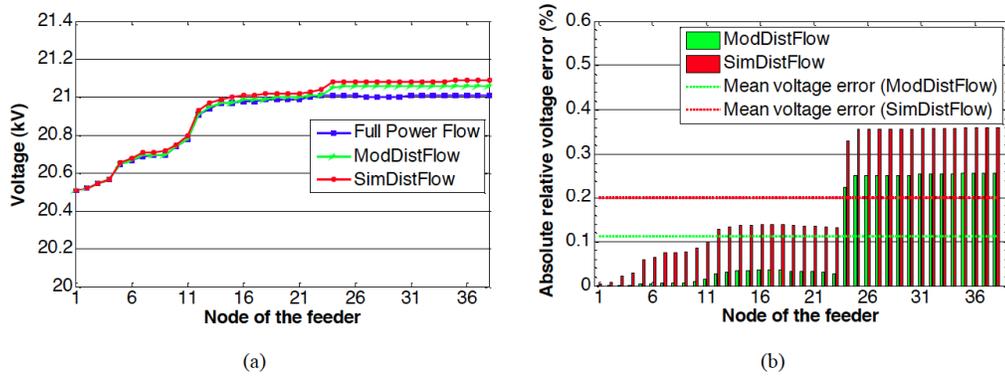
Figure 10: Load and installed DER capacity along the MV feeder.

during some hours of a day. This is indicatively shown for voltage calculations in Figure 12, where DER capacities are increased by 50% at all nodes. In this case, the accuracy in losses calculation improves by 4%.

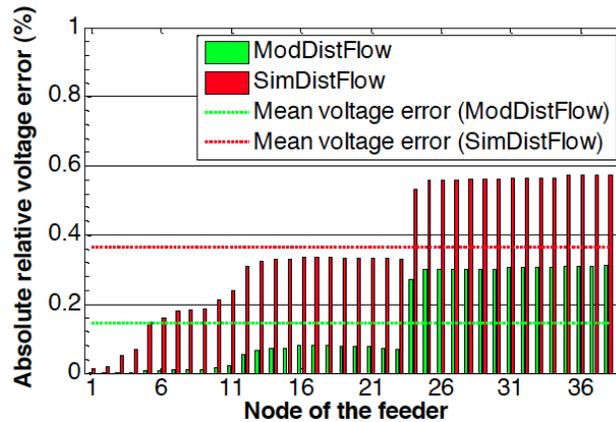
6. CONCLUSION

In this paper, a new distribution power flow method is proposed (*ModDistFlow*), derived from the simplified

equations of the *SimDistFlow* method after suitable modifications. The proposed method does not neglect active and reactive power losses, as is the case with other simplified methods commonly used in the literature, and incorporates them in the voltage calculations in a novel and intuitive manner which substantially enhances accuracy in voltage, power flows and losses calculations. Further, the *ModDistFlow* method is non-iterative and therefore



**Figure 11:** Implementation results for the MV feeder. **a)** Voltage profile along the feeder and **b)** Absolute relative voltage error (%) for each node.



**Figure 12:** Absolute relative voltage error (%) for each node for the MV feeder, assuming a 50% increase in installed DER capacity.

**Table 5: Feeder Data**

Branch Number	Conductor Type	Length (km)	Branch Number	Conductor Type	Length (km)
1	3x95 mm <sup>2</sup> ACSR	0.34	20	3x95 mm <sup>2</sup> ACSR	0.39
2	3x95 mm <sup>2</sup> ACSR	0.13	21	3x95 mm <sup>2</sup> ACSR	0.21
3	3x95 mm <sup>2</sup> ACSR	0.77	22	3x95 mm <sup>2</sup> ACSR	0.96
4	3x95 mm <sup>2</sup> ACSR	0.43	23	3x95 mm <sup>2</sup> ACSR	1.44
5	3x95 mm <sup>2</sup> ACSR	2.11	24	3x95 mm <sup>2</sup> ACSR	6.26
6	3x95 mm <sup>2</sup> ACSR	0.39	25	3x95 mm <sup>2</sup> ACSR	0.85
7	3x95 mm <sup>2</sup> ACSR	0.66	26	3x95 mm <sup>2</sup> ACSR	0.43
8	3x95 mm <sup>2</sup> ACSR	0.10	27	3x95 mm <sup>2</sup> Cu	2.47
9	3x95 mm <sup>2</sup> ACSR	0.12	28	3x95 mm <sup>2</sup> Cu	0.70
10	3x95 mm <sup>2</sup> ACSR	0.78	29	3x95 mm <sup>2</sup> Cu	0.45
11	3x95 mm <sup>2</sup> ACSR	1.11	30	3x95 mm <sup>2</sup> Cu	0.22
12	3x95 mm <sup>2</sup> ACSR	3.17	31	3x95 mm <sup>2</sup> Cu	3.14
13	3x95 mm <sup>2</sup> ACSR	0.91	32	3x95 mm <sup>2</sup> Cu	1.06
14	3x95 mm <sup>2</sup> ACSR	0.52	33	3x95 mm <sup>2</sup> Cu	0.36
15	3x95 mm <sup>2</sup> ACSR	0.01	34	3x95 mm <sup>2</sup> Cu	0.10
16	3x95 mm <sup>2</sup> ACSR	0.45	35	3x95 mm <sup>2</sup> Cu	1.89
17	3x95 mm <sup>2</sup> ACSR	0.10	36	3x95 mm <sup>2</sup> Cu	0.74
18	3x95 mm <sup>2</sup> ACSR	0.32	37	3x95 mm <sup>2</sup> Cu	1.82
19	3x95 mm <sup>2</sup> ACSR	1.16	38	3x95 mm <sup>2</sup> Cu	0.77

simple to implement and computationally efficient, while it is particularly suited for networks with high DER penetration levels.

The proposed method has been compared to the widely used *SimDistFlow* algorithm, already available in the literature, using both simplified and real world MV feeders as case studies and addressing a variety of loading, DER penetration conditions and feeder R/X ratios. *ModDistFlow* proved to be more accurate than *SimDistFlow*, the differences being magnified for heavily loaded feeders, high DER penetration levels and DER inductive power factor operation. On the other hand, when feeder load and DER output powers were relatively balanced all methods performed comparably, as the power flows on the network were reduced and therefore the effect of neglecting losses was not significant.

The substantial gains achieved by the proposed method in the calculation of voltages, power flows and losses, besides their self-evident value, eventually permit a more dependable evaluation of the DER hosting capacity of the network, more accurate estimation of HV/MV transformer losses and evaluation of reactive power compensation requirements at HV/MV substation level, as well as a more reliable calculation of the total active and reactive powers at the interface point with the HV system.

## NOMENCLATURE

### Abbreviations

DG = Distributed generation  
 DER = Distributed Energy Resources  
 HV = High voltage  
 MV = Medium voltage  
 OLTC = On-Load Tap Changer

### Indices

$g$  = Generation  
 $j$  = Node number; also respective branch number (branch  $j$  is the branch connecting nodes  $j$  and  $j+1$ )  
 $load$  = Load connected at a node  
 $loss$  = Losses on a branch

$nom$  = Nominal value  
 $r$  = Received by a node  
 $s$  = Sent by a node

### Parameters and Variables<sup>1</sup>

$z_j$  = Complex impedance ( $\Omega/\text{km}$ ) of branch  $j$   
 $r_j$  = Resistance ( $\Omega/\text{km}$ ) of branch  $j$   
 $x_j$  = Reactance ( $\Omega/\text{km}$ ) of branch  $j$   
 $S_{load_j}$  = Complex power of load connected at node  $j$   
 $p_{load_j}$  = Active power of load connected at node  $j$   
 $q_{load_j}$  = Reactive power of load connected at node  $j$   
 $I_j$  = Current on branch  $j$  (from node  $j$  to node  $j+1$ )  
 $S_{j,s}$  = Complex power flow at the sending end of node  $j$  (left end of branch  $j$ )  
 $P_{j,s}$  = Active power flow at the sending end of node  $j$  (left end of branch  $j$ )  
 $Q_{j,s}$  = Reactive power flow at the sending end of node  $j$  (left end of branch  $j$ )  
 $S_{j,r}$  = Complex power flow at the receiving end of node  $j+1$  (right end of branch  $j$ )  
 $P_{j,r}$  = Active power flow at the receiving end of node  $j+1$  (right end of branch  $j$ )  
 $Q_{j,r}$  = Reactive power flow at the receiving end of node  $j+1$  (right end of branch  $j$ )  
 $P_{loss_j}$  = Active power loss along branch  $j$   
 $Q_{loss_j}$  = Reactive power loss along branch  $j$   
 $V_j$  = Voltage at node  $j$   
 $V_{nom}$  = Nominal voltage

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<sup>1</sup>Power quantities in capital letters represent power flows on a branch; lowercase letters are used for node powers.

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