

Stability Analysis and Optimal State Feedback Control of Back-to-Back Converter

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Abstract: Developing of transmission systems and using of different energy sources to supply the demand of network power makes that the using of power electronic converters are increased. Among the power electronic converters, Back to Back (B2B) converters are operated in the power system today's widely. These converters can be located in connection of distributed power generation systems to power network and in HVDC transmission systems.

In this paper stability analysis and controllability of B2B converters based on the average model in HVDC power transmission systems is studied. Also, optimal state feedback control under different conditions for B2B converter is proposed. Finally, simulation results are illustrated in order to verify the capability of proposed control strategy.

Keywords: Back to Back converter, Stability, Controllability, Regulation, Tracking.

1. INTRODUCTION

Today's the use of Back to Back Converters in power systems is increasing [1-6]. The usages of these converters can be pointed to connecting the distributed power generation systems to power network and in HVDC transmission systems [1, 7, 8].

For example, in wind turbine systems, according to the changes in wind speed, power generation is fluctuated. One of the important problems that should be considered during the connection of wind turbine to main grid is proper synchronisation of this unit [2-5]. In this condition, B2B converters are used for synchronisation [2, 3]. Moreover, at the HVDC power transmission system, beginning of the line ac voltage convert to high voltage dc and then, the dc voltage convert to ac voltage again with desired voltage and frequency level at the end of the transmission line [1, 7, 8].

In all studies on the B2B converter, control of converter based on the real model has been considered. In [1] the steady state and dynamic performances of B2B converter based on voltage source converters during step changes of the active and reactive powers, balanced and unbalanced faults has investigated. In [2] converter control of wind turbine power generation system based on Optimal and Direct-Current Vector Control was studied. In [3] a control strategy is considered for direct-drive permanent-magnet wind-power generator using sensor less back-to-back PWM converter. Reference [4] studies control

strategy based on analytical strategies with aim to reducing destructive harmonics. In [5] converter control based on nonlinear decoupled control of back-to-back voltage source converter and in [6] limits of the neutral-point balance in back-to-back-connected three-level converters were studied. In [7] a 36 step voltage source converter that it's output voltage steps has increased, is proposed. This 36 steps b2b converter can control active and reactive power of HVDC systems independently. In [8] a fast optimum predictive control strategy for bipolar B2B converter is proposed. This paper will study on stability and controllability of B2B converter based on average model and then, by designing the optimal state feedback investigate the regulation and tracking under different condition of active and reactive powers. This paper is as follows:

Section 2 presents a complete description of the plant based on average model. Section 3 studies the stability and controllability of plant and then, in section 4 by designing the optimal state feedback improve the stability. Section 5 contains tracking analysis of proposed controller. In section 6 for validation of control strategy, simulation results are presented. Conclusion is drawn in section 7.

2. B2B CONVERTER MODEL

Figure 1 shows a simple model of B2B converter. It contains two power electronic converters. One of them works as a rectifier and converts the AC voltage to DC voltage and injects the power to DC link. Another converter works as an inverter and converts the DC voltage to AC form and then injects the power to AC grid through an inductance and a resistance per phase [2].

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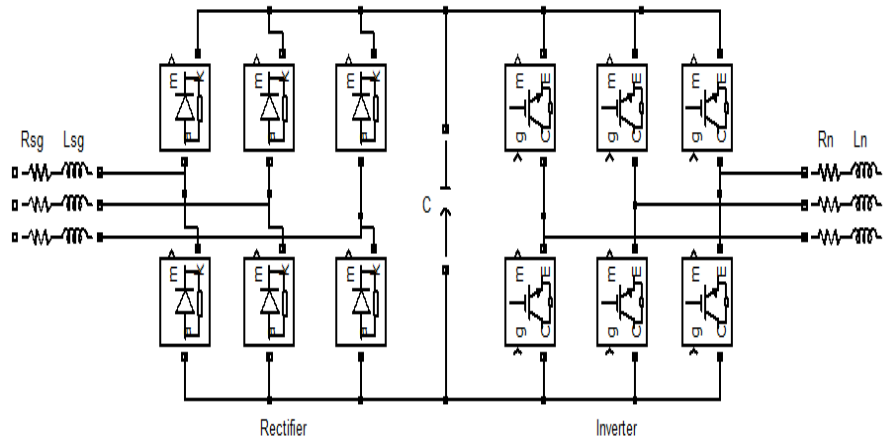


Figure 1: B2B Converter Circuit.

2.1. Rectifier Converter Model

The Equations that represent the behaviour of the rectifier in state space in dq frame follow from [2, 10, 11]:

$$\begin{cases} V_{sd} = -(R_s i_{sd} + L_d \frac{di_{sd}}{dt}) - w_e L_q i_{sq} \\ V_{sq} = (R_s i_{sq} + L_q \frac{di_{sq}}{dt}) + w_e L_d i_{sd} \end{cases} \quad (1)$$

$$\Rightarrow \begin{cases} i_{sd} = -\frac{R_s}{L_d} i_{sd} - \frac{w_e}{L_d} L_q i_{sq} - \frac{V_{sd}}{L_d} \\ i_{sq} = -\frac{R_s}{L_q} i_{sq} - \frac{w_e}{L_q} L_d i_{sd} - \frac{V_{sq}}{L_q} \end{cases}$$

Where R_s and L_d and L_q are the resistance and d-q axis inductance of rectifier side line respectively. V_{sd} and V_{sq} are d and q axis of rectifier side line.

2.2. Dynamic Model of Inverter

The proposed control strategy for inverter, have two main purposes [3, 12]:

1. Confidence of the quality of power injected into the network.
2. Minimizing the injected active and reactive power error.

Figure 2 shows inverter model [2, 11]. According to figure 2, the equations that represent the behaviour of the inverter in state space model [2, 10, 11].

$$\begin{cases} V_{d1} = R_f i_{gd} + L_f \frac{di_{gd}}{dt} - w_s L_f i_{gq} + V_d \\ V_{q1} = R_f i_{gq} + L_f \frac{di_{gq}}{dt} + w_s L_f i_{gd} + V_q \end{cases} \quad (2)$$

$$\Rightarrow \begin{cases} i_{gd} = -\frac{R_f}{L_f} i_{gd} + w_s i_{gq} - \frac{V_d}{L_f} + \frac{V_{d1}}{L_f} \\ i_{gq} = -\frac{R_f}{L_f} i_{gq} + w_s i_{gd} - \frac{V_q}{L_f} + \frac{V_{q1}}{L_f} \end{cases}$$

Where R_f and L_{gd} and L_{gq} are the resistance and d-q axis inductance of inverter side line respectively. V_{d1} and V_{q1} are d and q axis of inverter side line.

By combining equations (1) and (2), the equation that represents the behaviour of B2B converter in state space is obtained following:

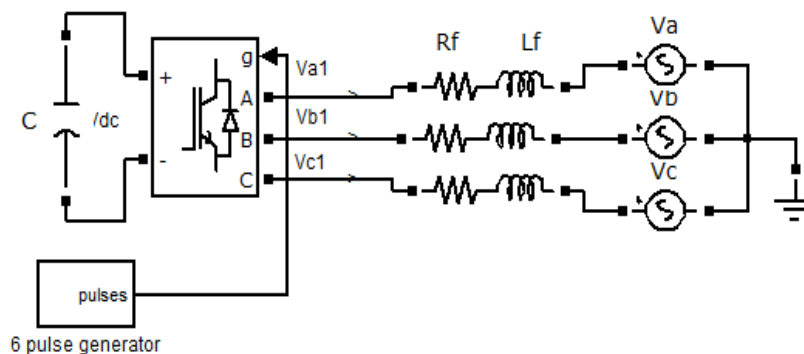


Figure 2: Inverter circuit.

$$\dot{X} = AX + BU + G$$

$$\begin{bmatrix} \dot{i}_{sd} \\ \dot{i}_{sq} \\ \dot{i}_{gd} \\ \dot{i}_{gq} \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_d} - \frac{w_e L_q}{L_d} & 0 & 0 & 0 \\ \frac{w_e L_d}{L_q} - \frac{R_s}{L_q} & 0 & 0 & 0 \\ 0 & 0 & -\frac{R_f}{L_f} & w_s \\ 0 & 0 & -w_s & -\frac{R_f}{L_f} \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{gd} \\ i_{gq} \end{bmatrix} \quad (3)$$

$$+ \begin{bmatrix} -\frac{1}{L_d} & 0 & 0 & 0 \\ 0 & -\frac{1}{L_q} & 0 & 0 \\ 0 & 0 & -\frac{1}{L_f} & 0 \\ 0 & 0 & 0 & -\frac{1}{L_f} \end{bmatrix} \begin{bmatrix} V_{sd} \\ V_{sq} \\ V_{gd} \\ V_{gq} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{V_{d1}}{L_f} \\ \frac{V_{q1}}{L_f} \end{bmatrix}$$

In Table 1 the parameters of system is presented.

Table 1: Parameters of the System

Voltage and frequency of grides	380v L-L F=50Hertz
$R_s = R_f$	0.04815ohm
$L_d = L_q = L_f$	mH

3. CONTROLLABILITY AND STABILITY ANALYSIS

3.1. System Controllability

The main purpose of system controllability is state feedback designing. According to equation (4), the necessary and sufficient condition for the system controllability is that the matrix w be full rank [3, 13, 14].

$$\begin{cases} w = [B \ AB \ A^2B \ \dots \ A^{n-1}B] \\ Rank(w) = n \end{cases} \quad (4)$$

In multiple-input systems, condition of controllability is [3]:

$$\det(w^T \times w) \neq 0 \quad (5)$$

According to equations (4) and (5):

$$\begin{cases} \det(w^T \times w) = -4.88e+013 \neq 0 \\ rank(w) = 4 \end{cases} \quad (6)$$

Therefore, the all states in model of B2B converter are controllable. So, state feedback control strategy could be designed for it.

3.2. System Stability

The stability is the most important characteristic for a control system. The stability of the closed loop system directly depends on root locus characteristic [3].

By using of the system eigen value, stability or instability of the system could be realized. If the eigen values locate in left side of imagine axis, it means that the system is stable, in otherwise, system is unstable.

Equation (7) is used to find the eigen values of the system [3, 13].

$$\det(\lambda I - A) = 0 \quad (7)$$

Where λ is the roots of Equation (7) that are eigen values. According to equations (3) and (7):

$$\begin{cases} \lambda_{1,2} = -20.93 \pm 314 i \\ \lambda_{3,4} = -20.93 \pm 314 i \end{cases} \quad (8)$$

The real part of eigen values are negative, so system is stable.

4. DESIGN OF OPTIMAL STATE FEEDBACK (LQR) CONTROL

Figure 3 shows block diagram of state feedback control which has been implemented in this paper.

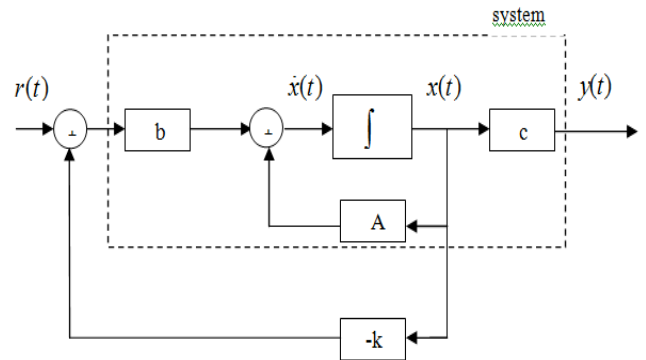


Figure 3: Block diagram of state feedback control.

In designing of optimal state feedback control, the control matrix k is calculated from equation (9) [3]:

$$k = R^{-1} B^T P \quad (9)$$

Where R is a positive definite matrix or a positive number, B is input matrix and P is calculated from rickety equations that are shown as follow:

$$A^T P + pA - PBR^{-1}B^T P + Q = 0 \tag{10}$$

$$\begin{cases} R = \rho I_n \\ Q = I_n \end{cases} \tag{11}$$

Where Q is a positive semi-definite matrix and n is number of state variables.

According to Equations (9) to (11), k is:

$$K = \begin{bmatrix} -0.95 & 0 & 0 & 0 \\ 0 & -0.95 & 0 & 0 \\ 0 & 0 & -0.95 & 0 \\ 0 & 0 & 0 & -0.95 \end{bmatrix} \tag{12}$$

The system's eigen values after implementation of state feedback control are achieved as follow:

$$\det(\lambda I - (A - BK)) = 0$$

$$\begin{cases} \lambda'_{1,2} = -435 \pm 314i \\ \lambda'_{3,4} = -435 \pm 314i \end{cases} \tag{13}$$

By comparing λ and λ' , it can be seen that the stability of the system with optimal state feedback control has been better than the previous condition.

5. REGULATION AND TRACKING ANALYSIS

In this section the Regulation and Tracking problems will be described.

5.1. Regulation Analysis

For analysing the properties of the state feedback controller, first the closed-loop system in state space should be considered [13].

$$\begin{cases} \Delta \dot{x} = A\Delta x + B\Delta u \\ \Delta y = C\Delta x \end{cases} \tag{14}$$

All state variables and outputs set at zero point or origin of coordinates. However, in most practical problems in control systems, setting the system outputs at nonzero constant values is desired. This subject can be achieved by a small change in implementation of state feedback controller as follows [1, 3, 7]:

Assume y_d shows the reference input, that $y_d \neq 0$.

First, the steady-state variables and inputs are calculated for achieving to y_d . Now suppose that there is an answer for system, and it can be displayed as follow [3]:

$$\begin{cases} x = M_x y_d \\ u = M_u y_d \end{cases} \tag{15}$$

Like non-linear equations, it is defined the incremental variables as following equations (16):

$$\begin{cases} \Delta x(t) = x(t) - x \\ \Delta u(t) = u(t) - u \\ \Delta y(t) = y(t) - y_d \end{cases} \tag{16}$$

In this condition, the mathematical equation makes that $y(t) = y_d$, therefore, incremental variable should be zero, $\Delta y = 0$, and this causes the Δx be zero. So:

$$\Delta x \rightarrow 0 \quad \Delta y \rightarrow 0 \Rightarrow y(t) \rightarrow y \tag{17}$$

For this purpose, the control signal is:

$$\Delta u = -k\Delta x \tag{18}$$

Where, by choosing a proper k , $A-Bk$ be internal stable, optimal steady values also will obtain. Hence in main system, controller is as:

$$\begin{aligned} u(t) &= u + \Delta u = u + k\Delta x \\ &= u + k(x - x) = (M_u + kM_x)y_d - kx \\ &= u_{ex} - kx \end{aligned} \tag{19}$$

After designing K and calculating M_x and M_k , it is not required using of incremental variables and the controller easily is implementable as shown in Figure 4 [3, 14].

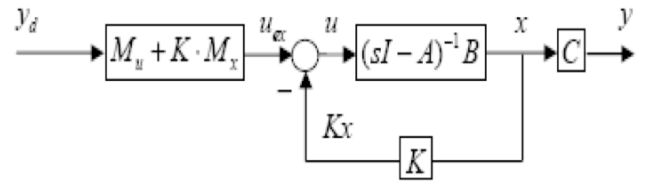


Figure 4: Implementing regulation controller without using incremental variables.

In Figure 4, $M_u + KM_x$ block is as feedback form. Then:

$$\Delta \dot{x} = (A - Bk)\Delta x$$

$$\begin{cases} x(t) = \Delta x(t) + x \\ u(t) = \Delta u(t) + u \\ y(t) = y_d + C\Delta x(t) \end{cases} \tag{19}$$

Therefore, for designing the state feedback controller, it needs to consider that the regulated state be set on zero. By choosing an appropriate k , the

matrix $A-Bk$ be stable, then, this controller can be implemented on main system [3].

5.2. Tracking Analysis

The final form of regulation problem could be easily generalized to tracking form. However, here $y_d = y_d(t)$ that is a function of time, so, the feedback loop must be updated.

6. SIMULATION AND RESULTS

In this section, the simulation results based on average model are presented.

Figures 4 and 5, represented the response of the system without/with state feedback respectively.

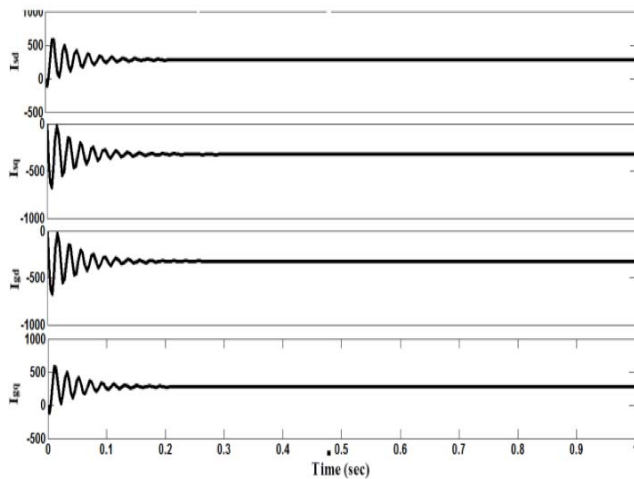


Figure 5: System response without state feedback.

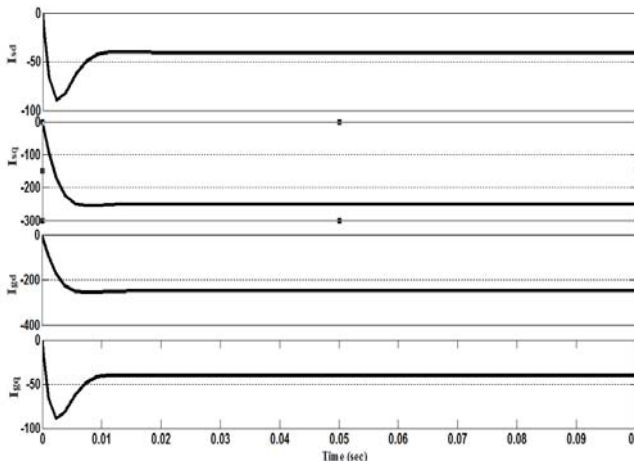


Figure 6: System response with state feedback.

According to Figures 5 and 6, it could be illustrated that, when state feedback controller is used, the state variables reach to steady state faster and transient state is reduces. After that, the regulation problem is considered. Thus, assume that injecting 50kw active

power and 10kvar reactive power is our regulation purpose. Using equation (21), it could be calculated the d and q references current in dq frame.

$$\begin{bmatrix} P \\ Q \end{bmatrix} = \frac{3}{2} \begin{bmatrix} V_d & V_q \\ V_q & -V_d \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} \tag{20}$$

Figure 7 show the reference currents in dq frame and figure 8 show the regulation result.

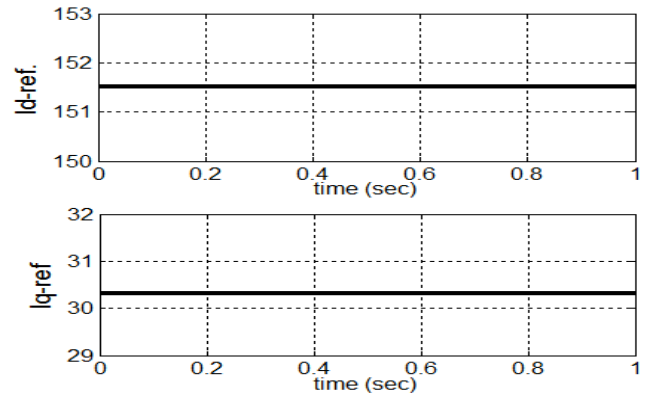


Figure 7: d-q axis reference currents.

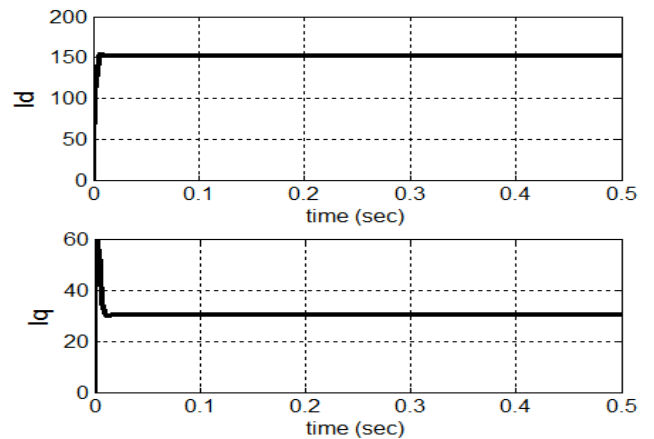


Figure 8: d-q axis currents regulation results.

According to Figures 7 and 8 and comparing them, it could be illustrated that regulation has been performed correctly.

6.3. Tracking

In this situation, active and reactive power injected to power system to tracking purpose, is function of time, as Figure 9. Using equation (21) calculating the reference currents in dq frame that are as Figure 10.

Figure 11 show the tracking simulation results. It can be seen that B2B converter tracking the performed correctly.

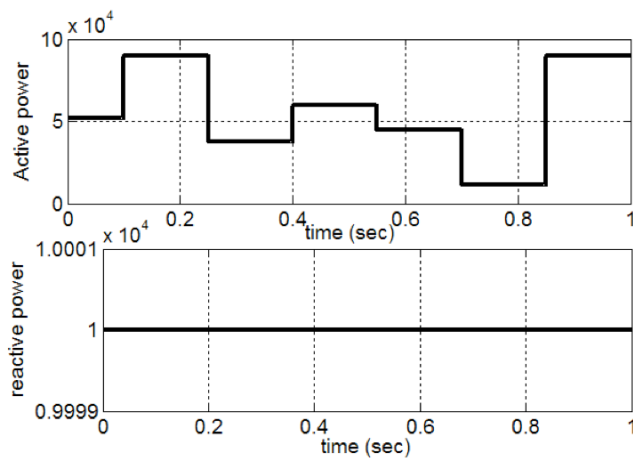


Figure 9: Active and reactive power injection to network.

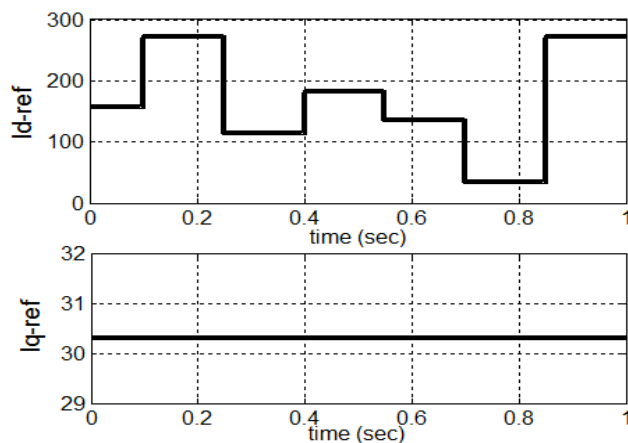


Figure 10: d-q axis current references.

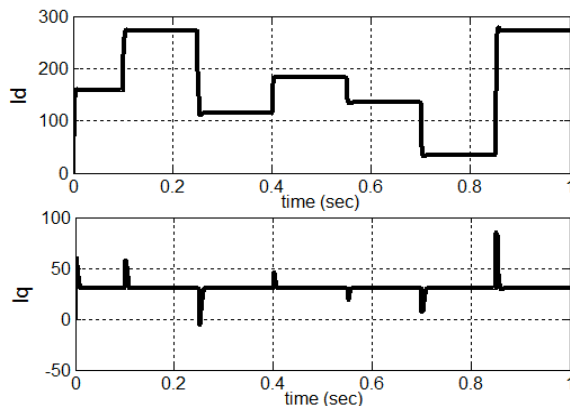


Figure 11: d-q axis current tracking results.

7. CONCLUSION

In this paper the stability and controllability of B2B converter based on average model have been studied.

Then, by designing optimal state feedback (LQR) controller, the stability of the system has been improved.

Finally, the Regulation and Tracking has been investigated. Simulation results show that with step changes in active and reactive powers, dq currents change respectively. Moreover, regulation and tracking problems have been performed correctly to verify the capability of proposed control strategy.

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